MPA International School

YEAR 9

MATHEMATICS

Reference Resources Booklet (1)

Name : ……………………………………………

ID : ……………………………………………

2026-2027 Academic Year

YEAR 9

MATHEMATICS

Reference Resources Booklet (1)

Unit – 1

Unit – 2

Unit – 3

Unit – 4

Preface

At MPA International School, we are committed to nurturing learners who are not only knowledgeable but also capable of guiding their own learning journey. This booklet has been carefully prepared to support our students as a confident, curious, and independent learner by providing clear, structured notes that reinforce key concepts and offer guidance across all related topics.

Each section in this booklet connects directly to the topics of the textbook, offering:

- Clear explanations of key ideas
- Concept summaries for quick revision
- Supportive notes that encourage **self-study** and **personal reflection**

This resource is designed to help students' **review at their own pace**, explore topics more deeply, and strengthen what they've learned in class. Whether they're preparing for a quiz, completing homework, or simply curious to know more—this booklet is here to guide and support them. Most importantly, use it to grow as a **self-directed learner**—someone who learns with purpose, confidence, and curiosity.

This resource is not meant to replace active learning or classroom discussion but to empower students to revisit important content at their own pace—whether reviewing after a lesson, preparing for a quiz, or exploring further out of curiosity.

We hope this booklet empowers you to take ownership of your learning with purpose and pride.

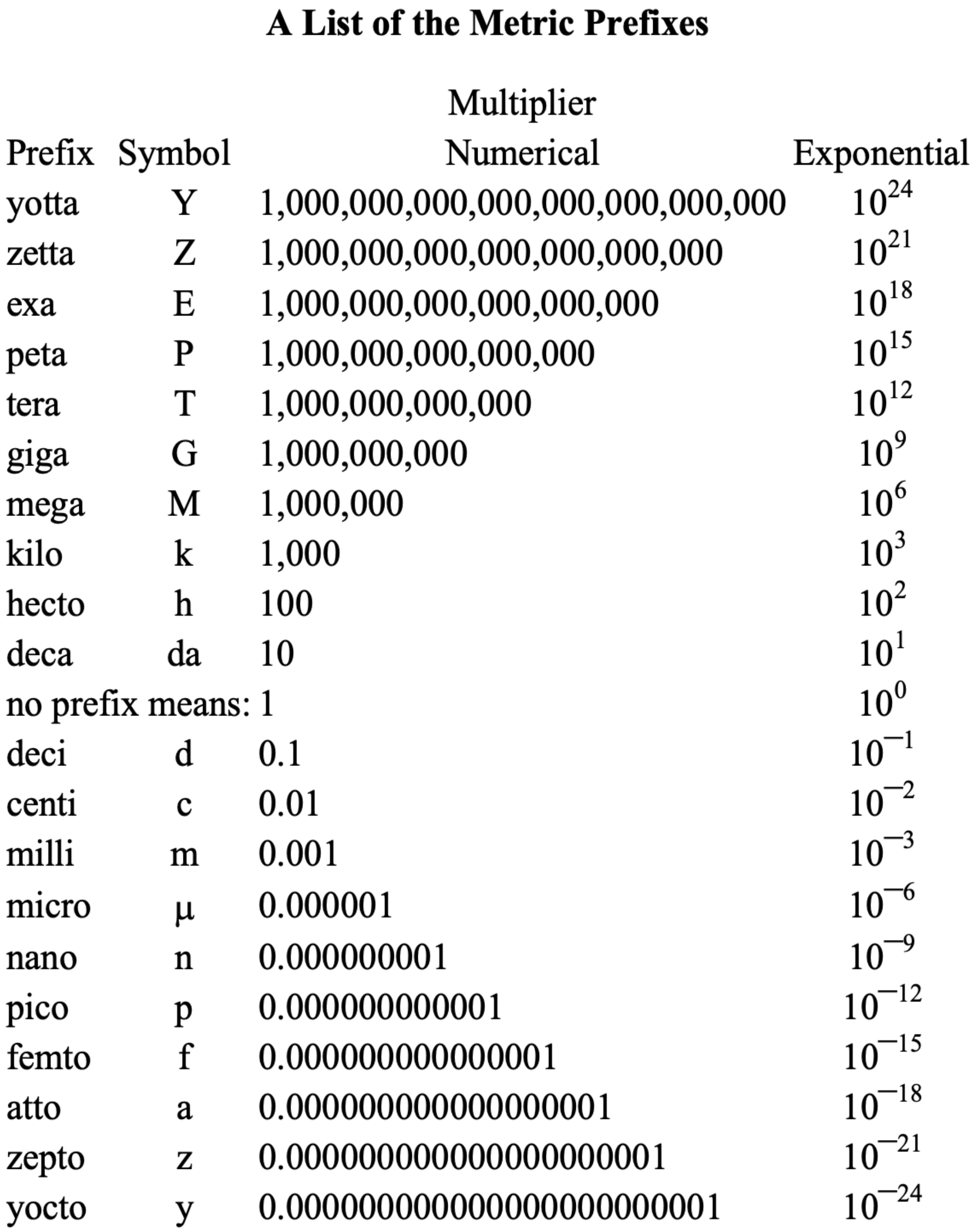
Academic Team
MPA International School

Contents

Unit (1) Significant figures, powers, and standard form

1.1 STEM: Powers of 10

Some powers of 10 have a name called a **prefix**. Each prefix is represented by a letter. The prefix for **10^6** is mega (**M**) as in **megabyte**.



A List of the Metric Prefixes

Prefix	Symbol	Multiplier	
		Numerical	Exponential
yotta	Y	1,000,000,000,000,000,000,000,000	10^{24}
zetta	Z	1,000,000,000,000,000,000,000	10^{21}
exa	E	1,000,000,000,000,000,000	10^{18}
peta	P	1,000,000,000,000,000	10^{15}
tera	T	1,000,000,000,000	10^{12}
giga	G	1,000,000,000	10^{9}
mega	M	1,000,000	10^{6}
kilo	k	1,000	10^{3}
hecto	h	100	10^{2}
deca	da	10	10^{1}
no prefix means:		1	10^{0}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	µ	0.000001	10^{-6}
nano	n	0.000000001	10^{-9}
pico	p	0.000000000001	10^{-12}
femto	f	0.000000000000001	10^{-15}
atto	a	0.000000000000000001	10^{-18}
zepto	z	0.000000000000000000001	10^{-21}
yocto	y	0.000000000000000000000001	10^{-24}

1.2 Calculating and estimating

Significant figures

What is a "significant figure?"

The number of significant figures in a result is the number of figures known with some degree of reliability.

The number 13.2 is said to have 3 significant figures. The number 13.20 is said to have 4 significant figures.

Rules for deciding the number of significant figures in a measured quantity:

(1) ***All nonzero digits*** are significant:
1.234 g has 4 significant figures,
1.2 g has 2 significant figures.

(2) ***Zeros between nonzero digits*** are significant:
1002 kg has 4 significant figures,
3.07 mL has 3 significant figures.

(3) ***Zeroes to the left of the first nonzero digits*** are not significant; such zeroes merely indicate the position of the decimal point:
0.001°C has only 1 significant figure,
0.012 g has 2 significant figures.

(4) ***Zeroes to the right of a decimal point in a number*** are significant:
0.023 mL has 2 significant figures,
0.200 g has 3 significant figures.

(5) ***When a number ends in zeros that are not to the right of a decimal point, the zeroes*** are not necessarily significant:
190 miles maybe 2 or 3 significant figures,
50,600 calories may be 3, 4, or 5 significant figures.

The potential ambiguity in the last rule can be avoided by the use of standard exponential, or "scientific," notation.

For example, depending on whether 3, 4, or 5 significant figures are correct, we could write 50,6000 calories as:

5.06×10^4 calories (3 significant figures)
5.060×10^4 calories (4 significant figures), or
5.0600×10^4 calories (5 significant figures).

Rules for rounding off numbers

(1) If the digit to be dropped is greater than 5, the last retained digit is increased by one.
For example,
12.6 is rounded to 13.

(2) If the digit to be dropped is less than 5, the last remaining digit is left as it is.
For example,
12.4 is rounded to 12.

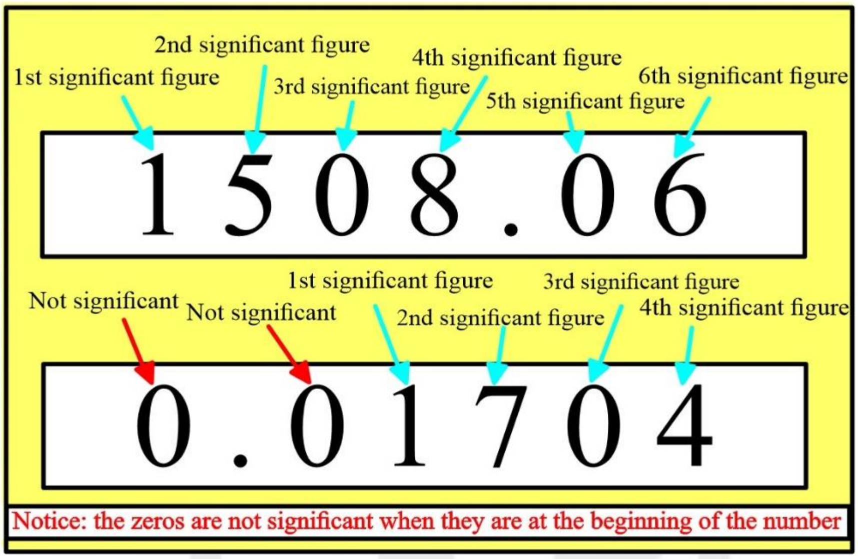
(3) If the digit to be dropped is 5, and if any digit following it is not zero, the last remaining digit is increased by one.
For example,
12.51 is rounded to 13.

(4) If the digit to be dropped is 5 and is followed only by zeroes, the last remaining digit is increased by one if it is odd but left as it is even. For example,

11.5 is rounded to 12,
12.5 is rounded to 12.

This rule means that if the digit to be dropped is 5 followed only by zeroes, the result is always rounded to the even digit.

The rationale is to avoid bias in rounding: half of the time we round up, and half the time we round down.



Notice: the zeros are not significant when they are at the beginning of the number

1.3 Indices

Index laws

Multiplication law: When multiplying with the same base (number/letter) we add the powers.

General rule: $\boldsymbol{a^m \times a^n = a^{m+n}}$

$2^5 \times 2^7 = 2^{5+7} = 2^{12}$ $x^3 \times x^8 = x^{3+8} = x^{11}$

When multiplying the terms we add the powers together.

Division law: When dividing with the same base (number/letter) we subtract the powers.

General rule: $\boldsymbol{a^m \div a^n = a^{m-n}}$

$2^{14} \div 2^7 = 2^{14-7} = 2^7$ $x^{10} \div x^8 = x^{10-8} = x^2$

When dividing the terms we subtract the powers together.

Brackets law: When raising a power to another power, we multiply the powers together.

General rule: $(a^m)^n = a^{mn}$

$(5^4)^2 = 5^{4 \times 2} = 5^8$ $(4h^9)^3 = 4^3 \times h^{9 \times 3} = 64h^{27}$

When raising to a power we multiply the powers together.

Key facts: You need to also remember that:

$$\mathrm{p} = p^1$$
$$p^0 = 1$$

Anything to the power zero is equal to 1.

Negative indices: A negative power performs the reciprocal.

General rule: $a^{-m} = \frac{1}{a^m}$

$3^{-1} = \frac{1}{3}$ $\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$ $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$ $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Fractional indices: The denominator of a fractional power acts as a "root.
The numerator of a fractional power acts as a normal power.

General rule: $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$ $\left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{\sqrt{25}}{\sqrt{16}}\right)^3 = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$

Changing bases: Index laws only work when the base numbers are the same, so sometimes it is necessary to change the base numbers using our knowledge of square and cube numbers.

$Write\ (4)^3\ as\ a\ power\ of\ 2$

$$4 = 2^2\ so\ (4)^3 = (2^2)^3 = 2^6$$

$Write\ (3)^5\ as\ a\ power\ of\ 9$

$$3 = \sqrt{9} = 9^{\frac{1}{2}}$$

$$so\ \ (3)^5 = \left(9^{\frac{1}{2}}\right)^5 = 9^{\frac{5}{2}}$$

1.4 Standard form

1.5 Calculating with Standard forms

Standard form

Standard form is a system of writing numbers. It is commonly used when dealing with very **large** and very **small** numbers, as it makes the numbers easier to read and use.

Standard form has the format $\boldsymbol{A \times 10^{n}}$.

For example: $2000 = 2 \times 10^{3}$

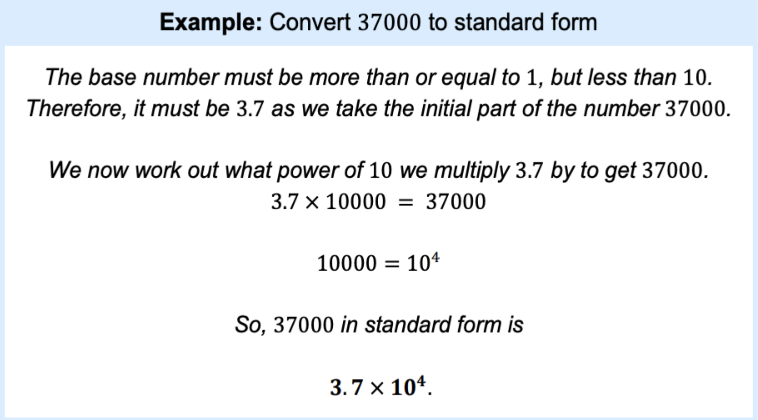
$0.0000045 = 4.5 \times 10^{-6}$

This is a more practical way of writing numbers such as 2000 and 0.0000045, respectively.

The first part of standard form ***A*,** is called the **base number**.

The base number must be **greater than or equal to 1**, and **less than 10**: $1 \leq A < 10$.

The second part of standard form is $\boldsymbol{10^{n}}$.
Whatever number *n* takes tells us what **power** of 10 we **multiply** the base number by.
The value of n must be an **integer** (a whole number).



Example: Convert 37000 to standard form

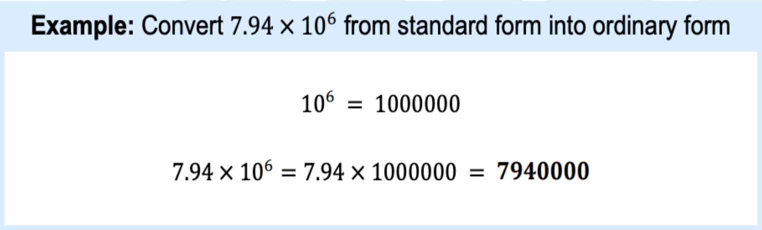
The base number must be more than or equal to 1*, but less than* 10*.*
Therefore, it must be 3.7 *as we take the initial part of the number* 37000*.*

We now work out what power of 10 *we multiply* 3.7 *by to get* 37000*.*
$$3.7 \times 10000 = 37000$$

$$10000 = 10^{4}$$

So, 37000 *in standard form is*

$$\boldsymbol{3.7 \times 10^{4}}.$$



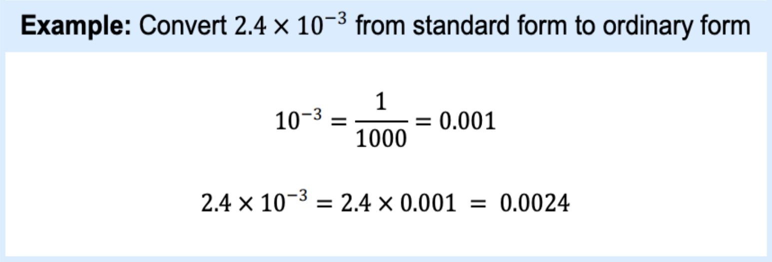
Example: Convert 7.94×10^{6} from standard form into ordinary form

$$10^{6} = 1000000$$

$$7.94 \times 10^{6} = 7.94 \times 1000000 = \mathbf{7940000}$$

The number that n takes, referring to the power of 10, can be **positive** or **negative**.

- Positive values of n mean that the actual number is usually **very large**.
- Negative values of n mean that the number is a **small decimal**. A negative value of n does not mean that the number is negative!



Example: Convert 2.4×10^{-3} from standard form to ordinary form

$$10^{-3} = \frac{1}{1000} = 0.001$$

$$2.4 \times 10^{-3} = 2.4 \times 0.001 = 0.0024$$

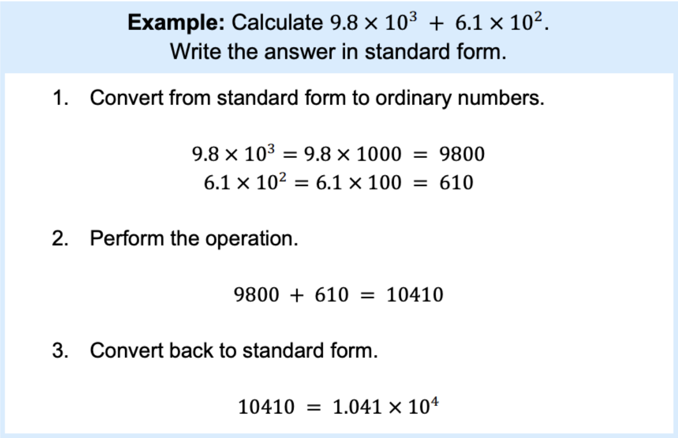
Calculations with Standard Form

Now that we can interpret standard form and convert both ways, we need to be able to perform operations with numbers in standard form.

Adding and Subtracting

When **adding** or **subtracting** two numbers that are both written in standard form, there are usually three steps:

(1) First, we must **convert** both numbers from standard form to normal form.
(2) We then **perform the operation** (addition or subtraction).
(3) Finally, if the question asks for the answer in standard form, we **convert** it back.



Example: Calculate $9.8 \times 10^3 + 6.1 \times 10^2$.
Write the answer in standard form.

1. Convert from standard form to ordinary numbers.

$$9.8 \times 10^3 = 9.8 \times 1000 = 9800$$
$$6.1 \times 10^2 = 6.1 \times 100 = 610$$

2. Perform the operation.

$$9800 + 610 = 10410$$

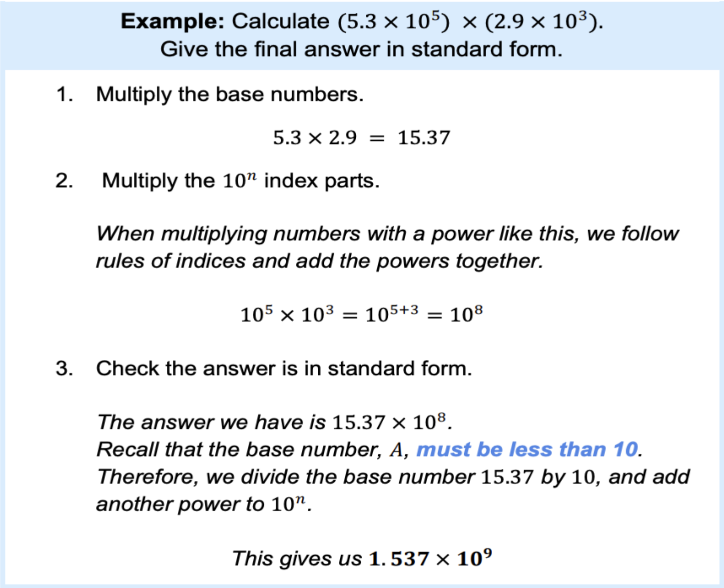
3. Convert back to standard form.

$$10410 = 1.041 \times 10^4$$

Multiplying and Dividing

Multiplying or dividing two numbers written in standard form is slightly different. We do not convert to an actual number. Again, there are three steps:

(1) Perform the operation (multiplication or division) on the base numbers (the part written as A).
(2) Perform the operation on the index $\mathbf{10^n}$ part.
(3) check that the final answer is still written in standard form (if the question requires it).



Example: Calculate $(5.3 \times 10^5) \times (2.9 \times 10^3)$.
Give the final answer in standard form.

1. Multiply the base numbers.

$$5.3 \times 2.9 = 15.37$$

2. Multiply the 10^n index parts.

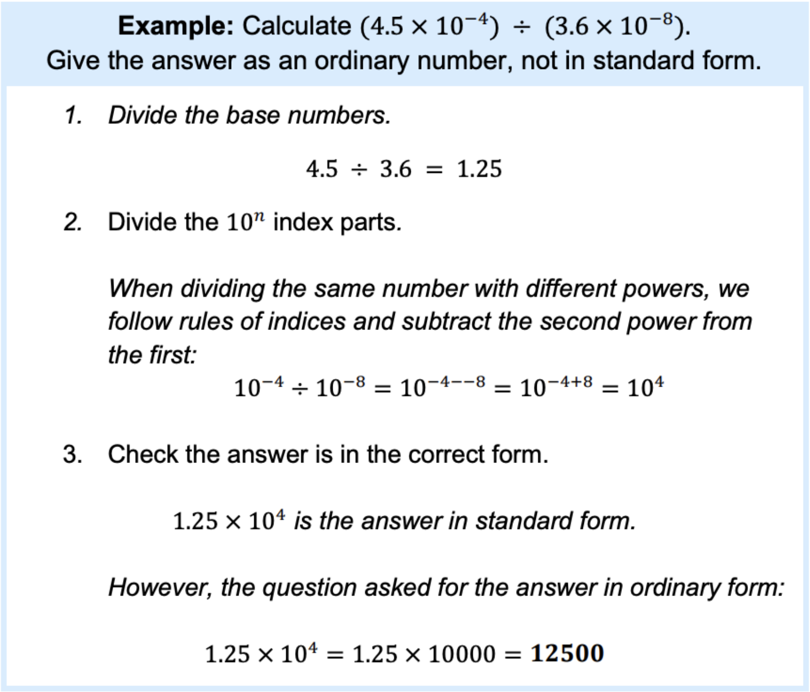
When multiplying numbers with a power like this, we follow rules of indices and add the powers together.

$$10^5 \times 10^3 = 10^{5+3} = 10^8$$

3. Check the answer is in standard form.

The answer we have is 15.37×10^8.
Recall that the base number, A, ***must be less than 10***.
Therefore, we divide the base number 15.37 *by* 10, *and add another power to* 10^n.

This gives us $\mathbf{1.537 \times 10^9}$



Example: Calculate $(4.5 \times 10^{-4}) \div (3.6 \times 10^{-8})$.
Give the answer as an ordinary number, not in standard form.

1. *Divide the base numbers.*

$$4.5 \div 3.6 = 1.25$$

2. Divide the 10^n index parts.

When dividing the same number with different powers, we follow rules of indices and subtract the second power from the first:

$$10^{-4} \div 10^{-8} = 10^{-4--8} = 10^{-4+8} = 10^4$$

3. Check the answer is in the correct form.

1.25×10^4 *is the answer in standard form.*

However, the question asked for the answer in ordinary form:

$$1.25 \times 10^4 = 1.25 \times 10000 = \mathbf{12500}$$

Key points to remember when using standard form:

- The base number, A, must be greater than or equal to 1, but less than 10: $1 \leq \boldsymbol{A} < 10$.
- n refers to the power of 10 and must be an integer.
- Negative values of n do not mean that the number is negative, just very small.

Explore

What units are used to measure distances in the universe?

What problems are there if you kilometres, or miles?

These need to be measured, which is possible when flying to the moon, but is different for greater distances.

Astronomical Units (AU, a.u. or ua) are often used for distances; 1 AU is the average distance between the Earth and the Sun as the Earth orbits the sun.

How could we measure large distances where we can't physically measure the distance?

Scientists measure the time that light takes to travel, or the relative movement of stars, to calculate distances.

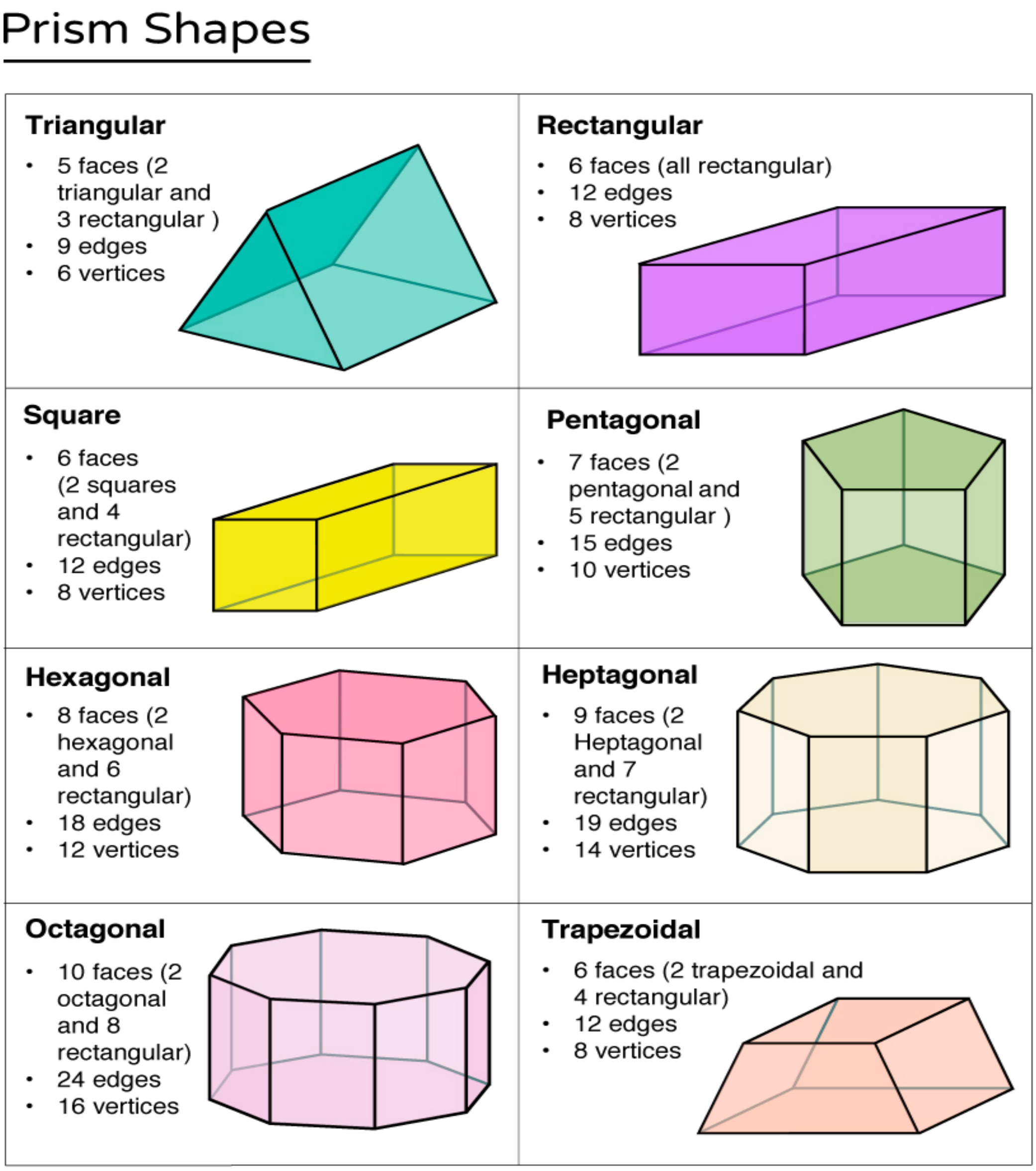
The **light-year (ly)** is the distance that light travels in one year (365.24 days); **kilolight-year (kly)** and **megalight-year (Mly)** are also used.

The **parsec (pc)** measures using the amount that a star moves as the Earth goes from one side of the Sun to the other; **kiloparsecs (kpc)** and **megaparsecs (Mpc)** are also used.

Unit (2) Significant figures, powers, and standard form

2.1 Surface area of prisms

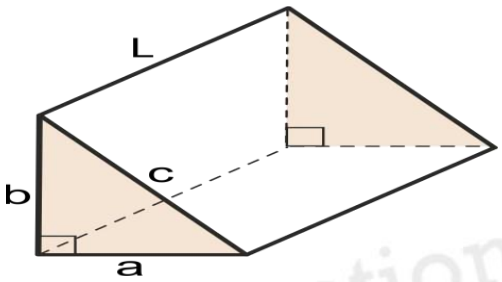
Prism: A **prism** is a solid with the same **cross-section** throughout its length.

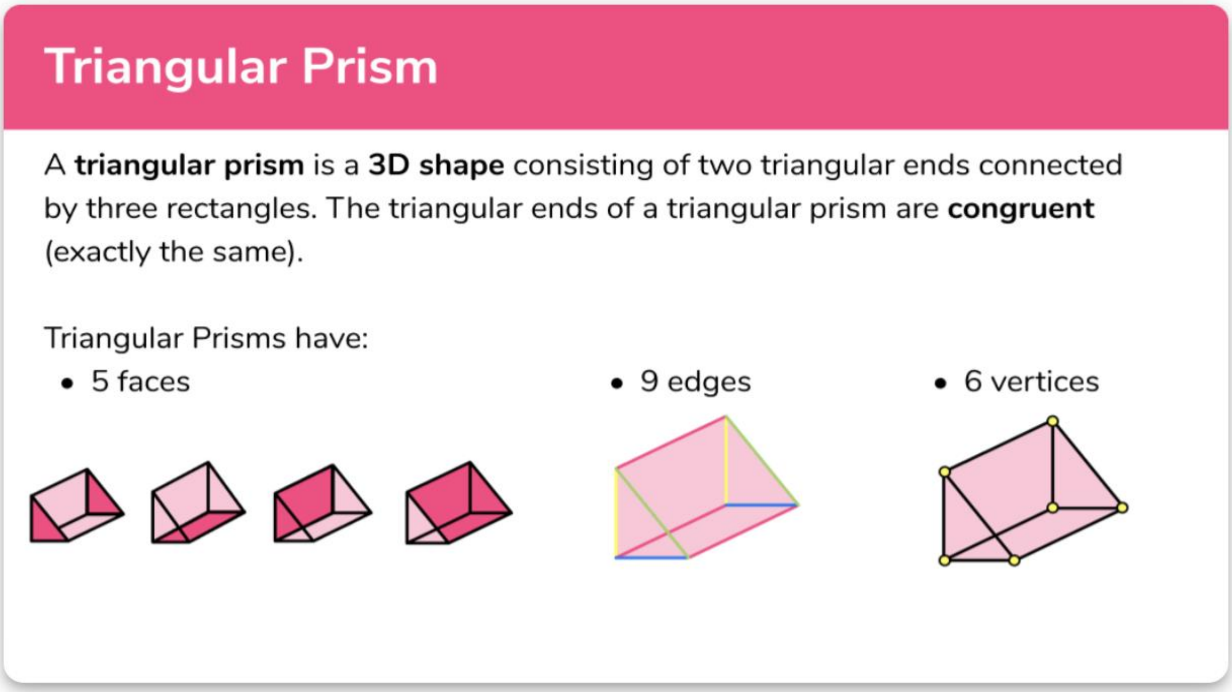


Prism Shapes

Triangular • 5 faces (2 triangular and 3 rectangular) • 9 edges • 6 vertices	**Rectangular** • 6 faces (all rectangular) • 12 edges • 8 vertices
Square • 6 faces (2 squares and 4 rectangular) • 12 edges • 8 vertices	**Pentagonal** • 7 faces (2 pentagonal and 5 rectangular) • 15 edges • 10 vertices
Hexagonal • 8 faces (2 hexagonal and 6 rectangular) • 18 edges • 12 vertices	**Heptagonal** • 9 faces (2 Heptagonal and 7 rectangular) • 19 edges • 14 vertices
Octagonal • 10 faces (2 octagonal and 8 rectangular) • 24 edges • 16 vertices	**Trapezoidal** • 6 faces (2 trapezoidal and 4 rectangular) • 12 edges • 8 vertices

Right prism: A **right prism** is a prism where the cross-section is at right angles to the length of the solid.





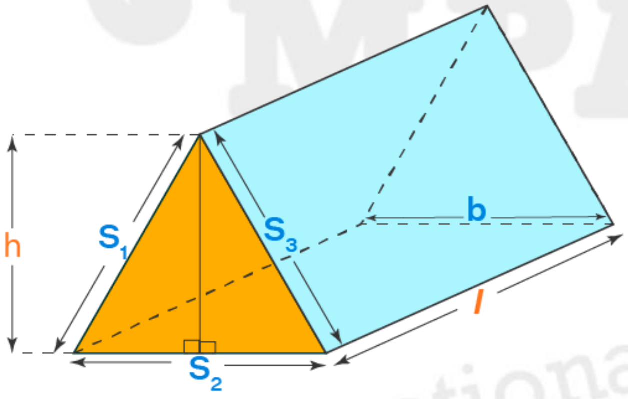
Triangular Prism

A **triangular prism** is a **3D shape** consisting of two triangular ends connected by three rectangles. The triangular ends of a triangular prism are **congruent** (exactly the same).

Triangular Prisms have:

- 5 faces
- 9 edges
- 6 vertices

Surface Area of a Triangular Prism



Total Surface Area = (Perimeter x Length) + (2 x Base Area)

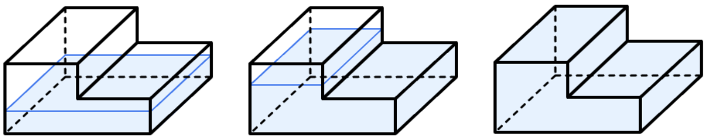
$$= (S_1 + S_2 + S_3)l + bh$$

2.2 Volume of prisms

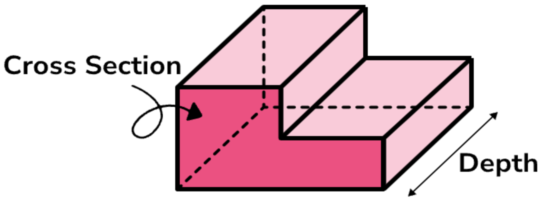
What is the volume of a prism?

The **volume of a prism** is how much space there is inside a prism.

Imagine filling this L-shaped prism fully with water. The total amount of water inside the prism would represent the volume of the prism in cubic units.



To calculate the volume of a prism, we find the **area of the cross section and multiply it by the depth**.



Volume of prism = Area of cross section x depth

How to calculate the volume of a prism

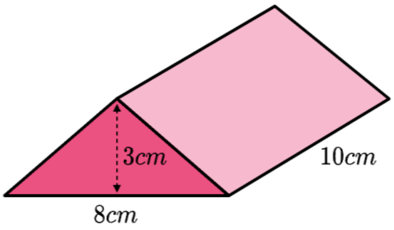
To calculate the volume of a prism:

- **Write down the formula.**
- **Calculate the area of the cross-section.**
- **Calculate the volume of the prism.**
- **Write the answer, including the units.**

Conversions of Units

1 *ml* = 1 cm^3
1 *l* = 1000 *ml* = 1000 cm^3
1000 *l* = 1 m^3
1 m^3 = 1 000 000 cm^3

Example 1: Volume of a triangular prism



1 Write down the formula.

Volume of prism = Area of cross section × depth

2 Calculate the area of the cross section.

$$\begin{aligned}\text{Area of triangle } &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 8 \times 3 \\ &= 12\end{aligned}$$

The area of the triangle is $12cm^2$.

3 Calculate the volume of the prism.

The depth of the prism is $10cm$.

$$\begin{aligned}\text{Volume of prism } &= \text{Area of cross section } \times \text{depth} \\ &= 12 \times 10 \\ &= 120\end{aligned}$$

4 Write the answer, including the units.

The measurements on this triangular prism are in centimetres so the volume will be measured in cubic centimetres.

Volume = $120cm^3$

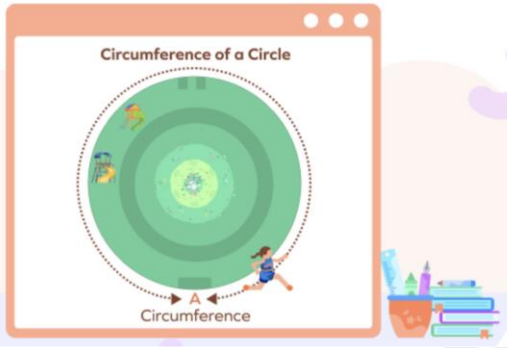
2.3 Circumference of a circle

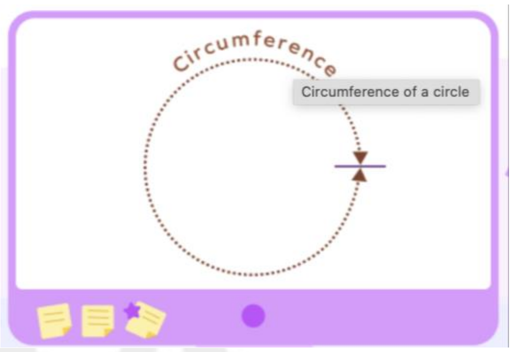
What is the Circumference of a Circle?

We see many circular objects daily, such as coins, buttons, wall clocks, wheels, etc. The boundary of any circular object has great significance in maths.

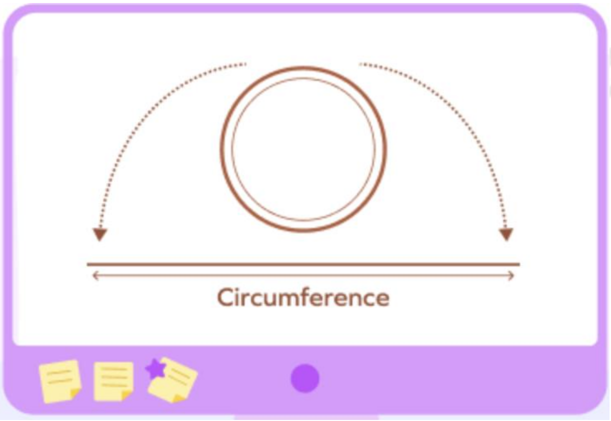
Circumference of a Circle: Definition

The circumference is the length of the boundary of a circle. It is also known as the “**perimeter**” of a circle. Since it represents length, it is measured in units of lengths such as feet, inches, centimetres, metres, miles, or kilometres.





If we cut open a circle and make a straight line, the length of the line would give us the circle’s circumference.

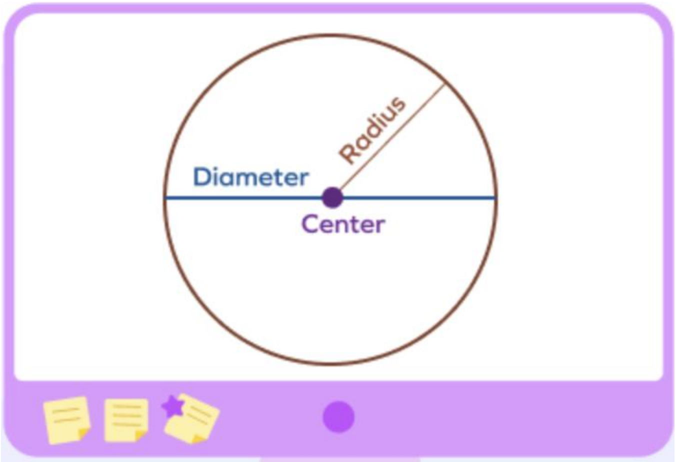


Diameter of the Circle

The **diameter** is a straight line passing through the centre that cuts the circle in half. Both its endpoints lie on the circumference of the circle.

Radius of the Circle

All points on the boundary of a circle are at an equal distance from its centre. The **radius** is the distance from the centre of the circle to any point on the **circumference of the circle**. It is half the length of the diameter.



Formula for the Circumference of a Circle

Circumference (C) / Diameter (d) = 3.14159 Or,

$$\frac{C}{d} = \pi$$

If we shift the diameter to the other side, we get:

C = **πd** … circumference of a circle using diameter

This gives us the **formula for the circumference of a circle** when the diameter is given.

Also, we know that the diameter of the circle is twice the radius or, **d = 2r**

So, replacing the value of d in the above formula, we get:

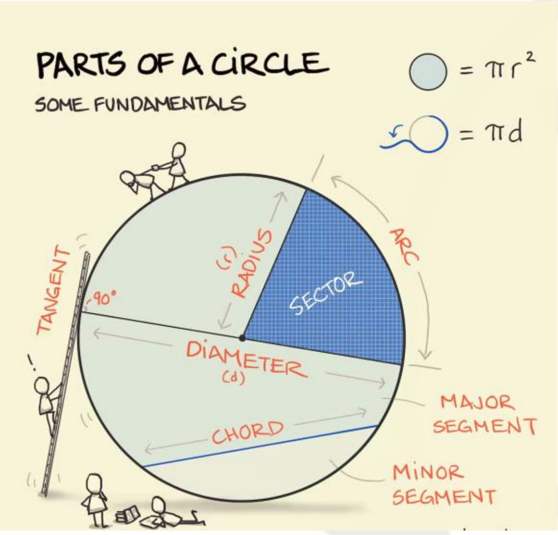
C = **π(2r)** **or C** = 2**πr** … circumference of a circle using radius

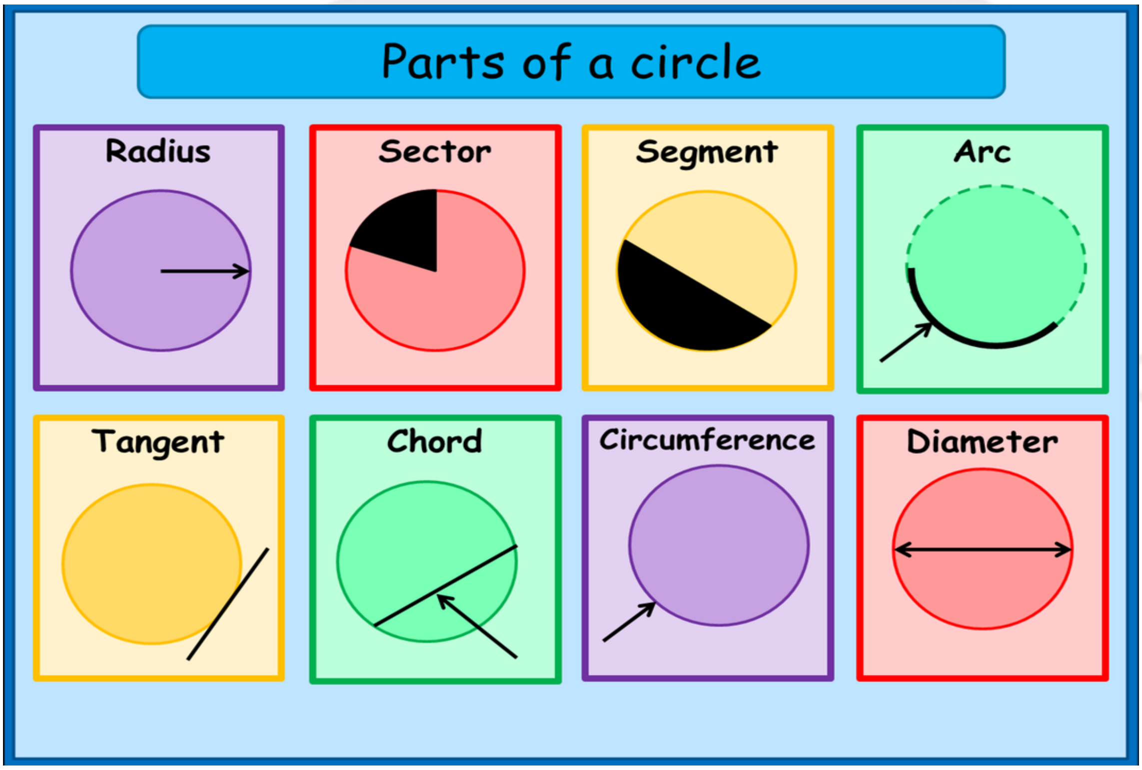
Therefore, the **circumference circle equation** is **C = 2πr**.

2.4 Area of a circle

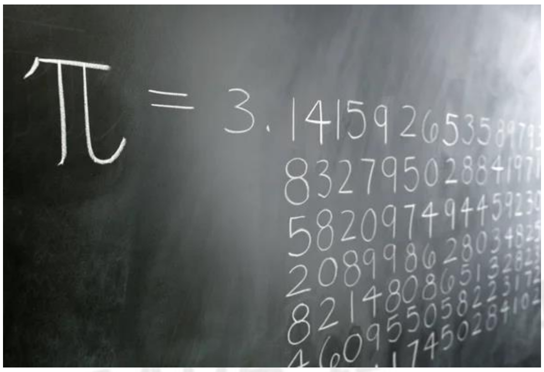
Parts of a circle

- **Diameter** — a line crossing the circle through its center. Also, the longest straight line you can draw in a circle.
- **Radius** — a line from the center to any point on the edge. Also, it is half the diameter.
- **Sector** — the classic Trivial Pursuit cheese, pizza, or cake slice formed by two radii.
- **Area** — equal to pi x the radius squared.
- **Circumference** — equal to pi x the diameter.
- **Arc** — a section of the circumference.
- **Chord** — a straight line touching any two points of a circle. The longest chord is the diameter.
- **Segment** — the areas on either side of a chord, major and minor.
- **Tangent** — a straight line just touching the edge of the circle without crossing it. It forms a right angle to the radius on the edge. Just as if you were able to lean a ladder against one.





What is Pi (π)?

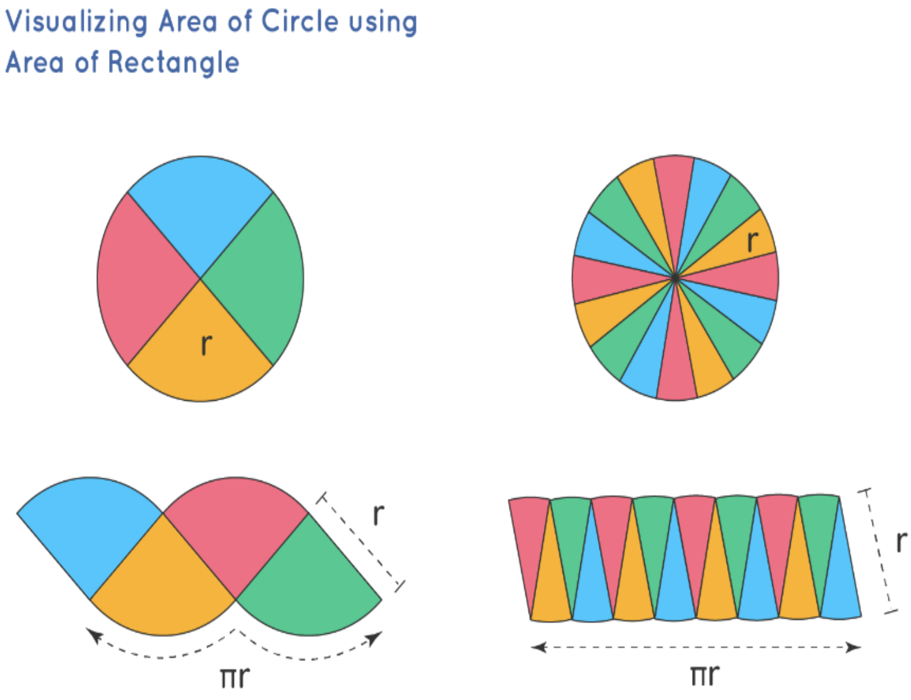


- **Pi**—which is written as the Greek letter for p, or **π**—is the ratio of the circumference of any circle to the diameter of that circle. Regardless of the circle's size, this ratio will always equal pi. In decimal form, the value of pi is approximately 3.14. (To only 18 decimal places, pi is 3.141592653589793238).

- Pi is most commonly used in certain computations regarding circles. Pi not only relates to circumference and diameter. Amazingly, it also connects the diameter or radius of a circle with the area of that circle by the formula: the area is equal to pi times the radius squared.

Formulation Derivation for "Area of a Circle"

Why is the area of the circle is **πr^2**? To understand this, let's first understand how the formula for the area of a circle is derived.



When the circle is divided into even smaller sectors, it gradually becomes the shape of a parallelogram. The more the number of sections it has more it tends to have the shape of a parallelogram as shown above.

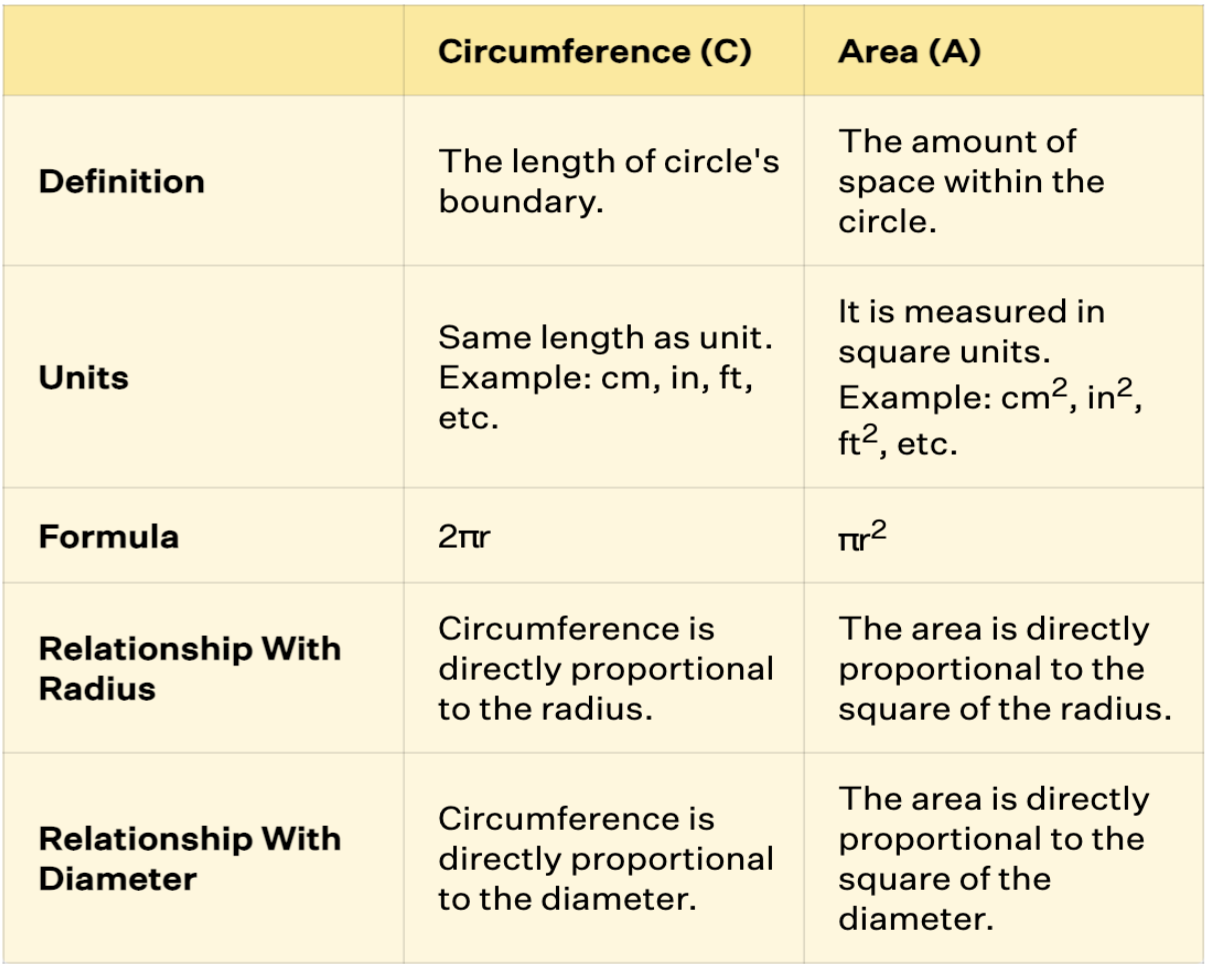
The area of a parallelogram is = base length × perpendicular height

The base length of a parallelogram = $\frac{1}{2}$ circumference of a circle (C)

The perpendicular height of a parallelogram = radius of a circle (r)

Thus,

$$\begin{aligned} \text{Area of a circle} &= \text{Area of a parallelogram} \\ &= \text{base length} \times \text{perpendicular height} \\ &= \tfrac{1}{2}\text{ circumference of a circle} \times \text{radius of a circle} \\ &= \tfrac{1}{2} \times 2\pi r \times r \\ \mathbf{A} &= \pi r^2 \end{aligned}$$

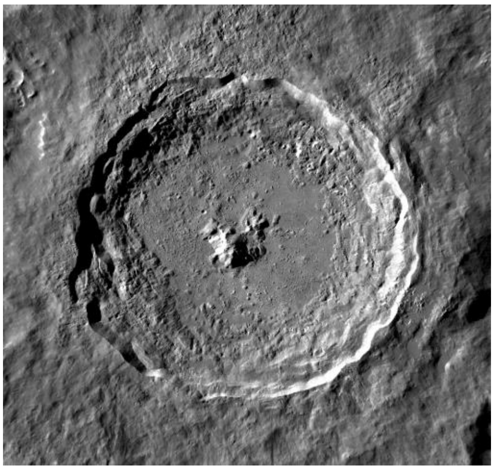


	Circumference (C)	**Area (A)**
Definition	The length of circle's boundary.	The amount of space within the circle.
Units	Same length as unit. Example: cm, in, ft, etc.	It is measured in square units. Example: cm^2, in^2, ft^2, etc.
Formula	$2\pi r$	πr^2
Relationship With Radius	Circumference is directly proportional to the radius.	The area is directly proportional to the square of the radius.
Relationship With Diameter	Circumference is directly proportional to the diameter.	The area is directly proportional to the square of the diameter.

<u>Fun facts about the Area of a circle</u>

- A circle is a flat, two-dimensional shape made up of a collection of points all the same distance from a central origin.
- No other shape with only one side can contain an area like a circle. A circle with an infinite diameter is a straight line.
- For as long as we've been around, we've been able to recognize circles. The Sun and Moon, the human eye, and shell shapes all form a circle because they are naturally occurring.
- The wheel (a circular object) was one of the most significant innovations in human history.
- The ratio of a circle's circumference to its diameter is Pi, an irrational number. Roughly speaking, it equates to 3.14.

- The area of a circle is infinite. It also has a straight line. You can make out some additional symmetry lines.
- The arc of a semicircle, which is half of a circle, is 180 degrees.
- It is mathematically proven that a circle will have no angles. You can find real-circle examples of circles in the form of flat plates, coins, and tires.



Source: NASA/Goddard/Arizona State University

Published: January 30, 2019

Tycho Crater is one of **the most prominent craters on the Moon**.

It appears as a bright spot in the southern highlands with rays of bright material that stretch across much of the nearside.

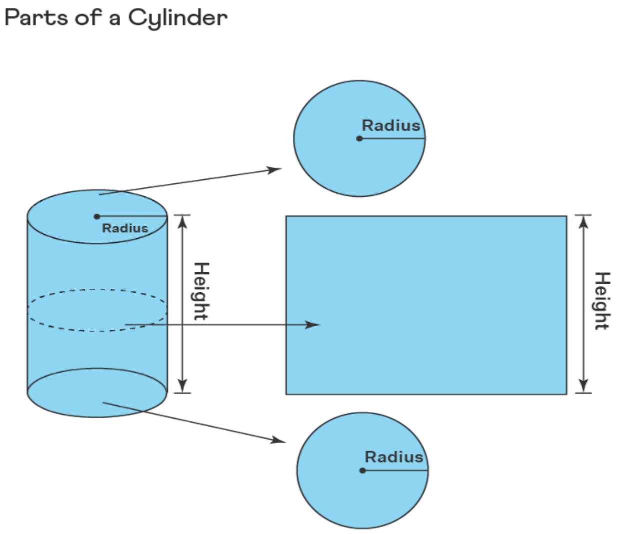
Tycho's prominence is not due to its size. It is just one among thousands of similar-sized craters at 53 miles (85 kilometers) in diameter.

2.5 Cylinder

Definition

In mathematics, a cylinder is a three-dimensional solid that holds two parallel bases joined by a curved surface at a fixed distance. These bases are normally circular (like a circle), and the center of the two bases is joined by a line segment called the axis. The perpendicular distance between the bases is the height, “h,” and the distance from the axis to the outer surface is the radius, “r,” of the cylinder.

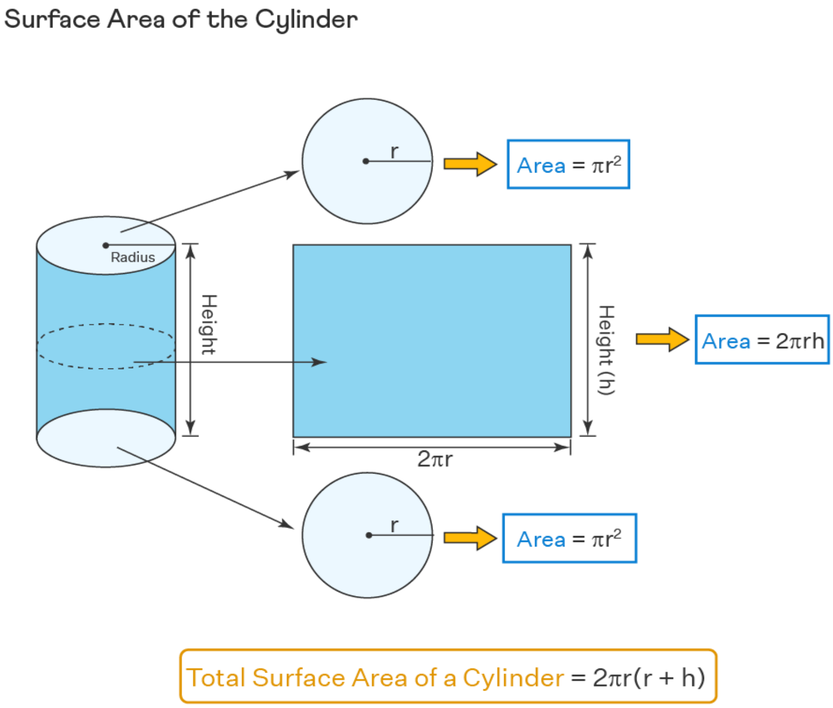
Below is the figure of the cylinder showing area and height.



Parts of a Cylinder

Total Surface Area of Cylinder

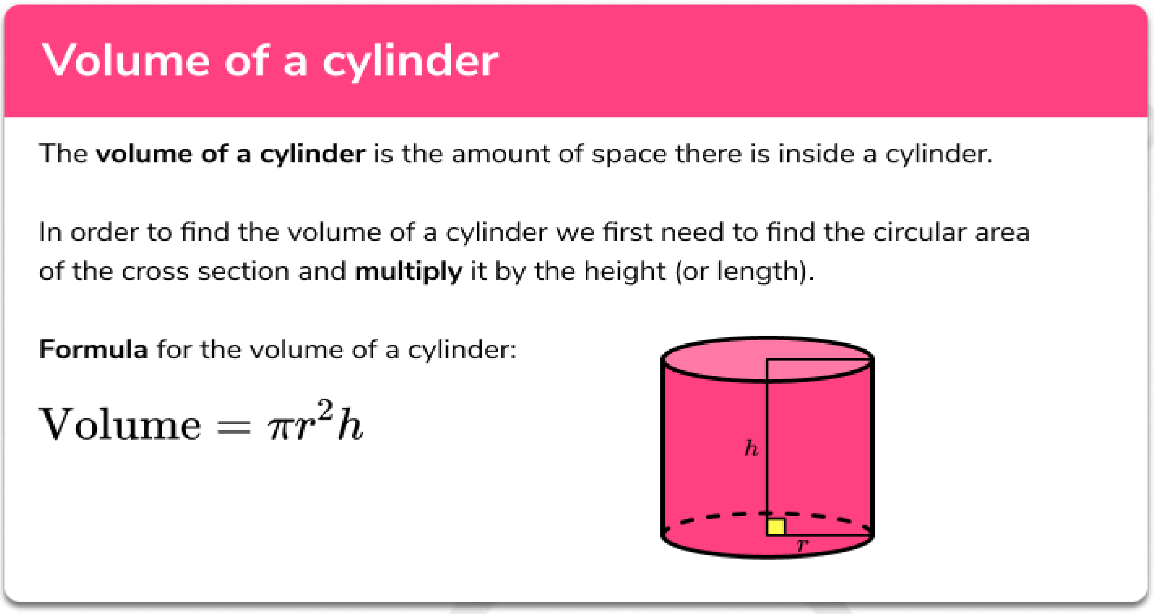
The area of any shape is the space occupied by it. A cylinder has 2 flat surfaces: circles and a curved surface that opens up as a rectangle. Consider the cylinder given below, whose height is 'h' and radius is 'r'. Let us open a cylinder in the 2-dimensional form and understand this.



Surface Area of the Cylinder

Total Surface Area of a Cylinder = $2\pi r(r + h)$

Volume of a Cylinder



Volume of a cylinder

The **volume of a cylinder** is the amount of space there is inside a cylinder.

In order to find the volume of a cylinder we first need to find the circular area of the cross section and **multiply** it by the height (or length).

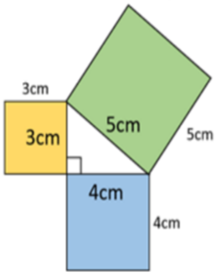
Formula for the volume of a cylinder:

$$\text{Volume} = \pi r^2 h$$

2.6 Pythagoras' Theorem

Year 9 Maths - Pythagoras

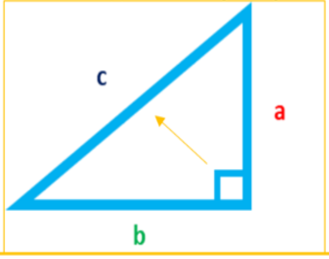
Pythagoras' theorem is an equation that describes a relationship between the 3 sides of a right-angle triangle. We can use it to determine a missing length when given the two other lengths.



The equation is:

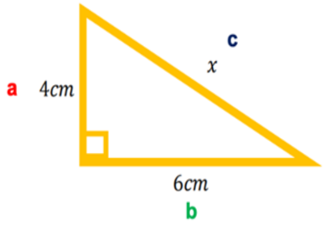
$a^2 + b^2 = c^2$

Where c is the hypotenuse and a and b are the two other sides. The hypotenuse is always the longest side of the triangle and can be found opposite the right angle.



Finding the length of hypotenuse

Example: Find the length of side x. Give your answer in 3 significant figures



You should always label the hypotenuse first. This is the side facing the right angle.

$a^2 + b^2 = c^2$

1) Substitute your values into the formulae:

$4^2 + 6^2 = x^2$

2) Work out the values that you can:

$16 + 36 = x^2$

$52 = x^2$

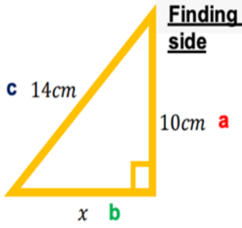
3) Now use inverse operations to find the values of x:

$x^2 = 52$

$x = \sqrt{52}$

$x = 7.211102551$ or 7.21 $(3\ s.f.)$

Finding the length of a shorter side



Example: Find the length of side x. Give your answer in 3 significant figures.

$a^2 + b^2 = c^2$

1) Substitute your values into the formulae:

$10^2 + x^2 = 14^2$

2) Work out the values that you can:

$100 + x^2 = 196$

3) Now use inverse operations to find the values of x:

$100 + x^2 = 196$

$-100 \qquad -100$

$x^2 = 96$

$\sqrt{} \qquad \sqrt{}$

$x = \sqrt{96}$

$x = 9.797958971cm$ or 9.8 $(3\ s.f.)$

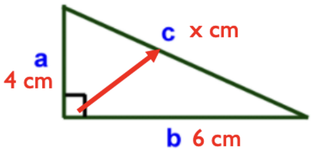
KEY VOCABULARY

Word	Definition
Hypotenuse	The longest side in a right angles triangle
Square number	The result when you multiply a number by itself
Right-angle triangle	A triangle in which one angle is of 90°
Square root	The reverse operation of squaring the number.

What you need to know:

Pythagoras' Theorem - Hypotenuse

You should always label the hypotenuse first.
This is the side facing the right angle.



This is surd form.
Sometimes you will be asked to leave your answer like this.

$a^2 + b^2 = c^2$

1) Substitute your values into the formulae:

$4^2 + 6^2 = x^2$

2) Work out the values that you can.

$16 + 36 = x^2$

$52 = x^2$

3) Now use inverse operations to isolate x.

$52 = x^2$

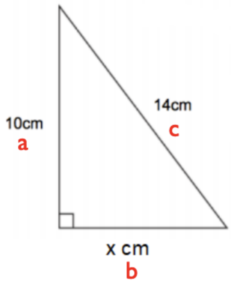
$(\sqrt{}) \quad (\sqrt{})$

$\sqrt{52} = x$

7.211102551 cm = x or 7.21 to 3 s.f

Pythagoras' Theorem – Shorter Sides

$a^2 + b^2 = c^2$



Sometimes you are asked to calculate the shorter sides, see below.

1) Substitute your values into the formulae:

$10^2 + x^2 = 14^2$

2) Work out the values that you can.

$100 + x^2 = 196$

3) Now use inverse operations to isolate x.

$100 + x^2 = 196$

$(-100) \qquad (-100)$

$x^2 = 96$

$(\sqrt{}) \quad (\sqrt{})$

$\sqrt{96} = x$

x = 9.797958971 cm or 9.80cm to 3 s.f

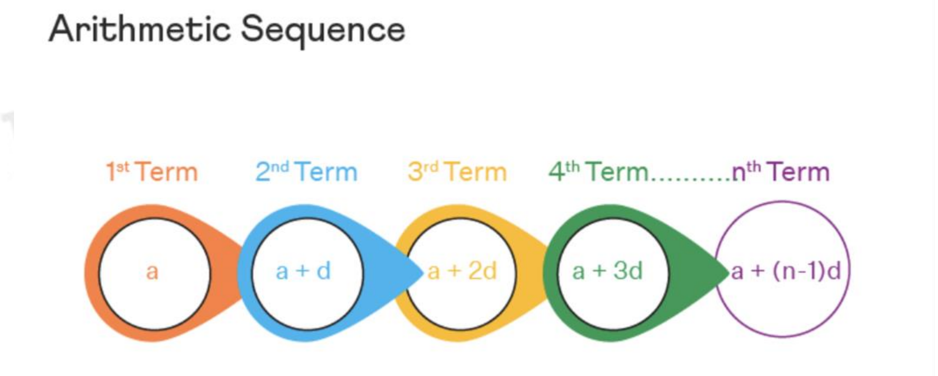
You need to get the numbers on one side, the x on it's own.
An extra step is needed.

Unit (3) Quadratics

3.1 Arithmetic and quadratic sequences

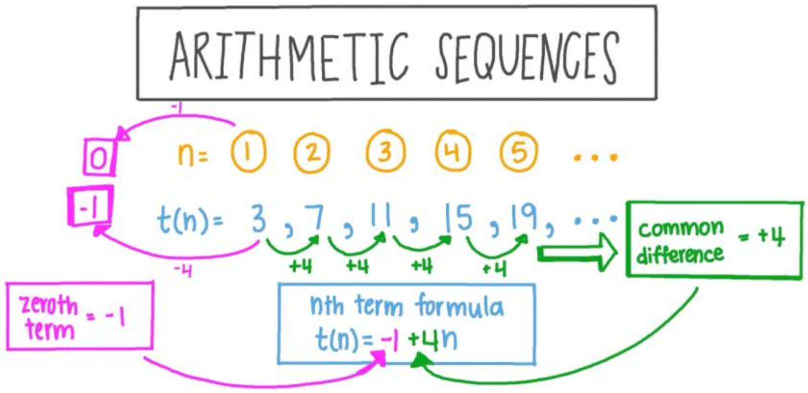
Arithmetic Sequence

An arithmetic sequence is a sequence of numbers in which each successive term is a sum of its preceding term and a fixed number. This fixed number is called a common difference. The terms of the arithmetic sequence are of the form a, a+d, a+2d, ...



Example: Mushi put $30 in her piggy bank when she was 7 years old. She increased the amount on each successive birthday by $3. So, the amount in her piggy bank follows the pattern of $30, $33, $36, and so on. The succeeding terms are obtained by adding a fixed number, that is, $3. This fixed number is called a common difference. It can be positive, negative, or zero.

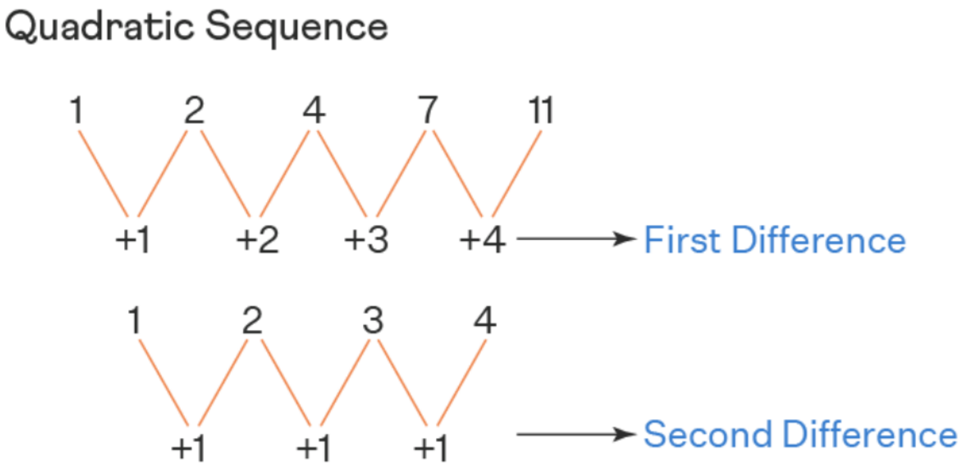
#In an arithmetic sequence, each successive term is obtained by adding the common difference to its preceding term.

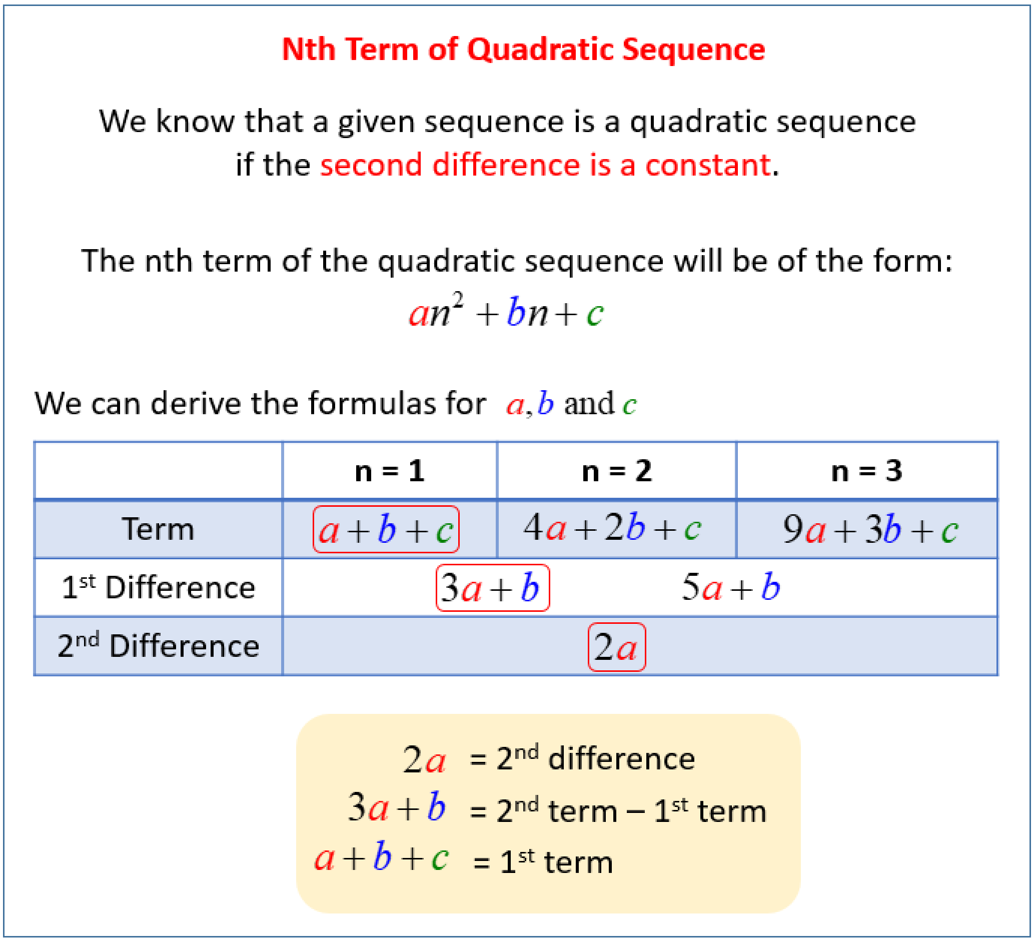


Quadratic Sequence

We have already seen that if the differences (referred to as first differences) between every two successive terms are the same, then it is called an arithmetic sequence (which is also known as a linear sequence). But if the first differences are NOT the same, and instead, the second differences are the same, then the sequence is known as a quadratic sequence.

Example: The sequence 1, 2, 4, 7, 11, ... is a quadratic sequence because its second differences are the same. Take a look at the figure below.





Nth Term of Quadratic Sequence

We know that a given sequence is a quadratic sequence if the second difference is a constant.

The nth term of the quadratic sequence will be of the form:

$$an^2 + bn + c$$

We can derive the formulas for a, b and c

	n = 1	n = 2	n = 3
Term	$\boxed{a+b+c}$	$4a+2b+c$	$9a+3b+c$
1st Difference	$\boxed{3a+b}$	$5a+b$	
2nd Difference		$\boxed{2a}$	

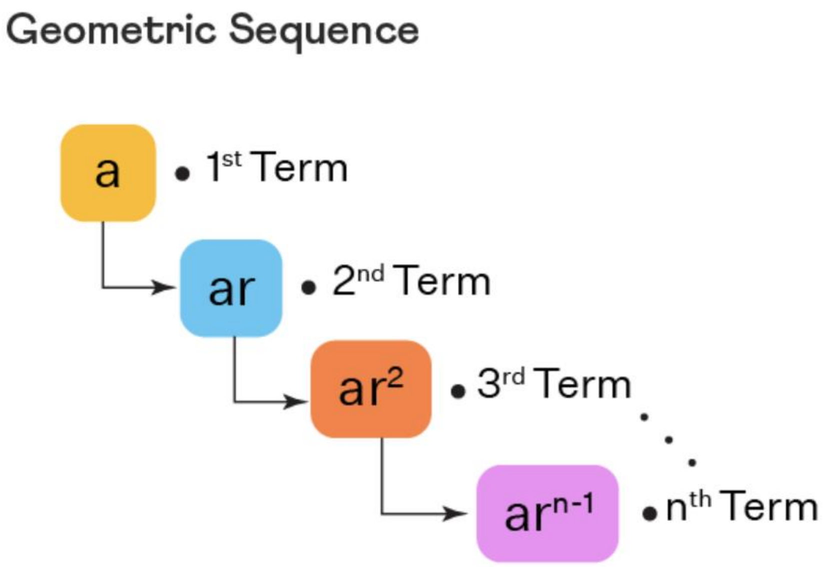
$2a$ = 2nd difference

$3a+b$ = 2nd term – 1st term

$a+b+c$ = 1st term

3.2 Geometric sequences

A geometric sequence is a sequence where every term bears a constant ratio to its preceding term. This ratio is called the "common ratio". The terms of the geometric sequence are of the form a, ar, ar^2, ...



#In a geometric sequence, each successive term is obtained by multiplying the common ratio to its preceding term.

nth Term of an Arithmetic Sequence

$$a_n = a_1 + (n - 1)d$$

nth Term of a Geometric Sequence

$$a_n = a_1 r^{n-1}$$

where,

a_n = n^{th} term

a_1 = The first term of the sequence

n = term position

d = common difference

r = common ratio

3.3 Expanding

What Does "Expanding" Mean?

Expanding means multiplying out expressions inside brackets to write them in a simplified form without parentheses.

For example: $(x + 3)\ (x + 2) \rightarrow x^2 + 5x + 6$

Multiplying Two Brackets (Double Brackets)

Method: **FOIL** or Area Grid

Example: (x+3) (x+4)

Using FOIL:

- **F (First)**: $x\,x = x^2$
- **O (Outer)**: $x \cdot 4 = 4x$
- **I (Inner)**: $3\,x = 3x$
- **L (Last)**: $3 \cdot 4 = 12$

Add them together: $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

Using an Area Grid (Visual Tool):

	x	4
x	$\mathbf{x^2}$	$\mathbf{4x}$
3	$\mathbf{3x}$	**12**

Total: $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

Squaring a Linear Expression

$$(x + a)^2 = (x + a)(x + a)$$

Example: $(x + 5)^2 = (x + 5)\ (x + 5) = x^2 + 10x + 25$

□Tip: The middle term is $2 * x * a$

Quadratic Identities (Special Patterns)

These help you expand quickly and reverse them when factorising.

□ Identity 1: $(a + b)^2 = a^2 + 2ab + b^2$

- Example: $(3x + 2)^2 = 9x^2 + 12x + 4$

□ Identity 2: $(a - b)^2 = a^2 - 2ab + b^2$

- Example: $(2x - 5)^2 = 4x^2 - 20x + 25$

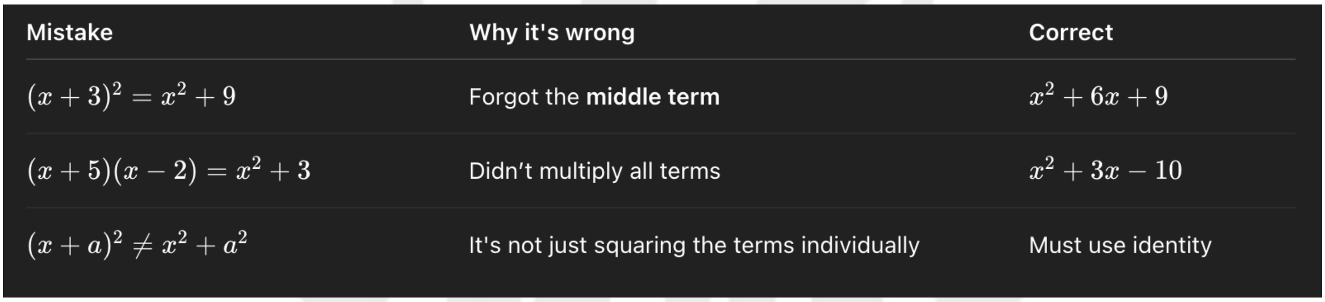
□ Identity 3: Difference of Squares

$$(a+b)(a-b) = a^2 - b^2$$

- Example: $(x + 6)(x - 6) = x^2 - 36$

□ These identities are useful shortcuts in both expanding and factorising.

□ Common Misconceptions



Mistake	Why it's wrong	Correct
$(x+3)^2 = x^2 + 9$	Forgot the **middle term**	$x^2 + 6x + 9$
$(x+5)(x-2) = x^2 + 3$	Didn't multiply all terms	$x^2 + 3x - 10$
$(x+a)^2 \neq x^2 + a^2$	It's not just squaring the terms individually	Must use identity

□ Visual Summary

Diagram idea: A triangle showing the flow:

- **Start**: Brackets
 → **Expand using FOIL / Identity**
 → **Simplify**
 → **Quadratic Expression**

You can also use a **flowchart** showing when to use FOIL, identity, or the grid method.

3.4 Factorising

□ Key Vocabulary

Term	Definition
Quadratic Equation	An equation involving x^2 (e.g., $ax^2 + bx + c = 0$).
Factorising	Rewriting an expression as a product of two binomials (brackets).
Solution	The value(s) of x that satisfy the equation.
Roots	The values of x that make the equation true (also called solutions).
Factor	An expression that divides evenly into another expression.
Zero Product Property	If the product of two factors is zero, then at least one of the factors must be zero.

□ Key Knowledge

1. Standard Form of a Quadratic Equation

A quadratic equation is written as: $ax^2 + bx + c = 0$.

where a, b, and c are constants, and x is the unknown.

2. Steps for Solving Quadratic Equations by Factorising

- **Write the equation in standard form**: Ensure the equation is equal to zero.

Example: $x^2 + 7x + 12 = 0$

- **Factor the quadratic expression**: Find two numbers that multiply to give c and add to give b.

Example: For $x^2 + 7x + 12 = 0$, find two numbers that multiply to 12 and add to 7

(These are 3 and 4).

The factorised form is: $(x+3)(x+4) = 0$

- **Use the Zero Product Property**

If $(x + 3)(x + 4) = 0$, set each factor equal to zero: $x + 3 = 0$ or $x + 4 = 0$.

- **Solve for x**: $x = -3$ or $x = -4$

3. Types of Quadratic Equations

- **Simple Quadratic** (e.g., $x^2 + 7x + 12 = 0$)
- **Difference of Squares** (e.g., $x^2 - 9 = 0$)

  Factorised as: $(x - 3)(x + 3) = 0$.

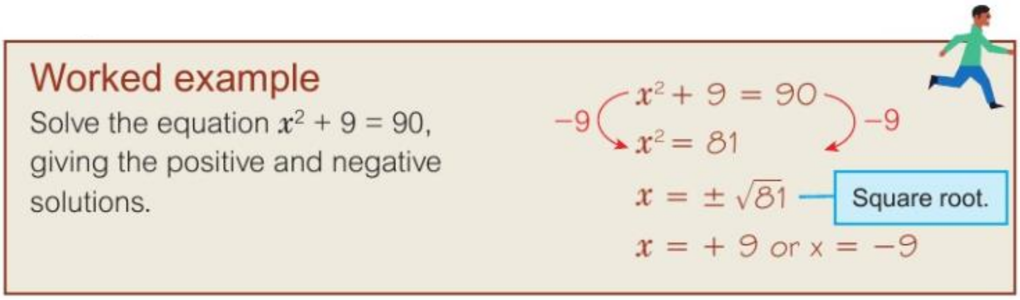
□ Common Mistakes

- Forgetting to set each factor equal to zero.
- Incorrectly factorising the quadratic expression.
- Not checking the solutions by substituting them back into the original equation.

□ Tips for Success

- Always check your factorisation by expanding the brackets.
- Ensure the equation is set to zero before factorising.
- Be familiar with common factorisation patterns, such as the difference of squares.

3.5 Solving Quadratic Equations



Worked example

Solve the equation $x^2 + 9 = 90$, giving the positive and negative solutions.

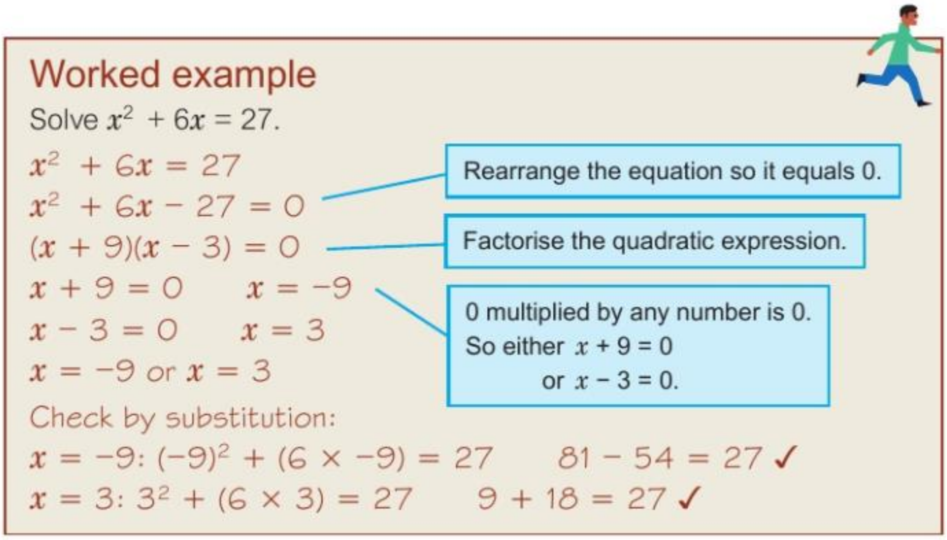
$x^2 + 9 = 90$

-9 -9

$x^2 = 81$

$x = \pm\sqrt{81}$ Square root.

$x = +9$ or $x = -9$



Worked example

Solve $x^2 + 6x = 27$.

$x^2 + 6x = 27$

$x^2 + 6x - 27 = 0$ Rearrange the equation so it equals 0.

$(x + 9)(x - 3) = 0$ Factorise the quadratic expression.

$x + 9 = 0 \qquad x = -9$

$x - 3 = 0 \qquad x = 3$

$x = -9$ or $x = 3$

0 multiplied by any number is 0.
So either $x + 9 = 0$
or $x - 3 = 0$.

Check by substitution:

$x = -9$: $(-9)^2 + (6 \times -9) = 27 \qquad 81 - 54 = 27$ ✓

$x = 3$: $3^2 + (6 \times 3) = 27 \qquad 9 + 18 = 27$ ✓

Unit (4) Constructions

4.1 Constructing shapes

Key Vocabulary

Term	Definitions
Net	A 2D shape that can be folded to form a 3D solid.
Face	A flat surface of a 3D shape.
Edge	A line where two faces meet.
Vertex	A corner where edges meet.
Compass	A tool used to draw arcs and circles.
Perpendicular	At 90° to a given line or surface.
Bisect	To divide into two equal parts.

Essential Skills and Methods

1. Drawing Nets of 3D Shapes

Solids	Description of net
Cube	6 equal squares arranged in a cross shape.
Cuboid	6 rectangles (opposite faces equal).
Triangular Prism	2 identical triangles + 3 rectangles.
Square-based Pyramid	1 square base + 4 triangular faces.

✏ **Tip:** Use a ruler to measure accurately and draw straight lines. Label sides and check symmetry before cutting or folding.

2. Constructing Triangles (Given Different Information)

- **SSS (Side-Side-Side):**

1. Draw the longest side using a ruler.
2. Use a compass to draw arcs from each end of the line, using the other two sides as radii.
3. The point where arcs meet is the third vertex.

- **SAS (Side-Angle-Side):**

1. Draw one side with a ruler.
2. Use a protractor to draw the given angle.
3. Use a compass to mark the length of the second side.

- **ASA (Angle-Side-Angle):**

1. Draw the given side.
2. At each end, draw the given angles using a protractor.
3. Extend the angle lines until they meet.

➡ **Tip:** Always label your triangle with the correct letters and mark all known sides and angles.

3. Constructing Nets Using a Ruler and Compasses

- Begin by drawing one face of the 3D shape (e.g., the base of a cube).
- Use a compass to copy lengths and ensure equal sides.
- Draw adjoining faces using accurate angles (usually 90°).
- Use a protractor or compass for right angles and symmetry.
- Label all faces and fold lines.

□ **Tip:** A net should fold perfectly into the 3D shape. Check the number and shape of faces.

Diagrams to Include

(These can be drawn or printed for the organizer.)

- Net of a cube
- Net of a square-based pyramid
- Triangle construction using SSS
- Compass arc examples
- Correct use of ruler, protractor, and compass

Key Facts to Remember

- Use **sharp pencils** for precision.
- **Label** all sides and angles clearly.
- Always **check measurements** twice before drawing.
- **Practice constructions** step-by-step before drawing final versions.
- Understanding **3D shapes** helps with spatial awareness and problem-solving.

4.2 Constructions 1

Key Constructions

1. Bisecting a Line Segment

Steps:

1. Place the compass on one end of the line segment (point A).
2. Draw an arc above and below the line.
3. Repeat from the other end without changing the compass width (point B).
4. The arcs intersect above and below the line – draw a straight line through the intersections.
5. This line bisects AB at 90° and marks the midpoint.

2. Constructing a Perpendicular Line

a) From a Point on a Line:

1. Place the compass point on the point (P) and draw arcs cutting the line at two points on either side.
2. From each of those new points, draw arcs above or below the line (must cross).
3. Draw a line from P through the intersection of the arcs. This is perpendicular to the original line.

b) From a Point off a Line:

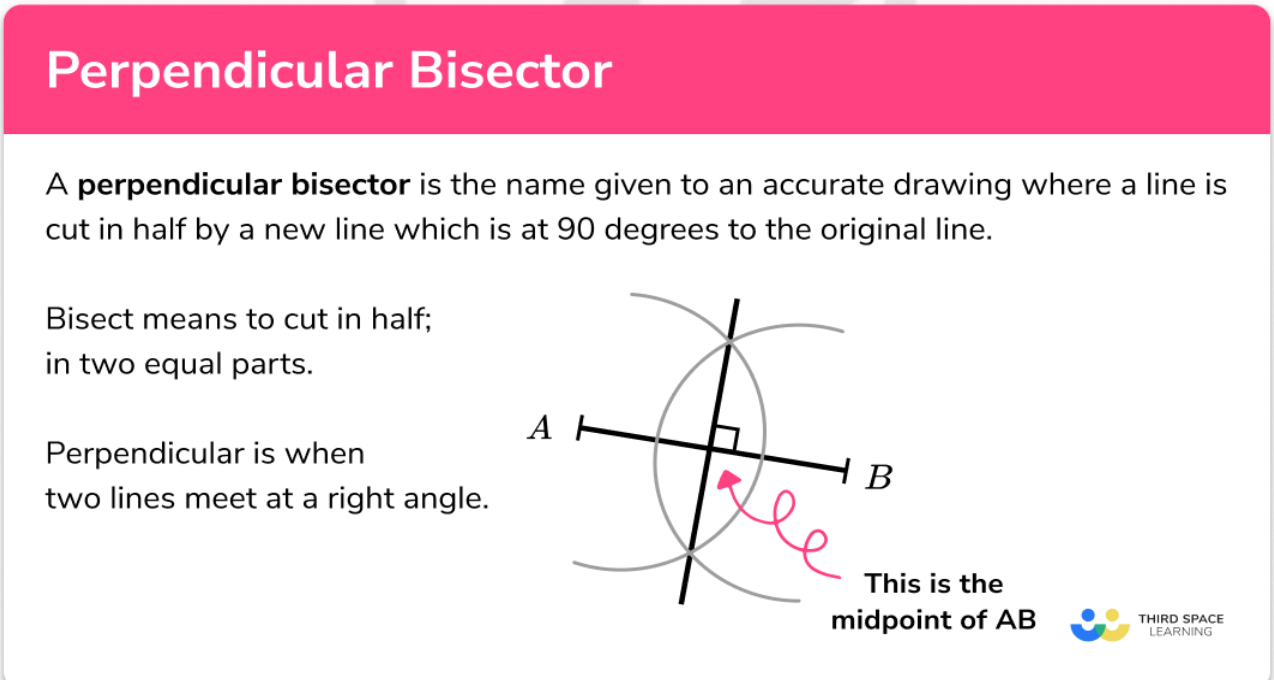
1. Place the compass point on the off-line point (P) and draw an arc that crosses the line in two places.
2. From the two intersection points, draw two arcs below the line so they intersect.
3. Draw a line from P through the arc intersection. This line is perpendicular to the original line.

Key Vocabulary

Word	Definition
Bisect	To divide something into equal parts
Perpendicular	Two lines meeting at a right angle (90°)
Arc	A part of a circle's edge
Intersection	The point where lines or arcs cross
Compass	A drawing tool for circles and arcs

Common Mistakes

- Changing the compass width halfway through a construction
- Not making arcs long enough to intersect
- Using a protractor instead of a compass for these tasks



Perpendicular Bisector

A **perpendicular bisector** is the name given to an accurate drawing where a line is cut in half by a new line which is at 90 degrees to the original line.

Bisect means to cut in half;
in two equal parts.

Perpendicular is when
two lines meet at a right angle.

This is the
midpoint of AB

4.3 Constructions 2

1. Bisecting an Angle with a Compass and Ruler:

- Step 1: Place the compass on the vertex of the angle.
- Step 2: Draw an arc that crosses both arms of the angle.
- Step 3: Mark the points where the arc intersects the angle's arms.
- Step 4: With the compass at each intersection point, draw two arcs that intersect inside the angle.
- Step 5: Draw a straight line from the vertex to the intersection of the arcs. This is the angle bisector.

2. Drawing Accurate Diagrams:

- Use a sharp pencil, compass, and ruler.
- Measure lengths and angles precisely.
- Label all key points clearly (e.g., A, B, C).
- Keep the construction neat and clear.

Example Problem:

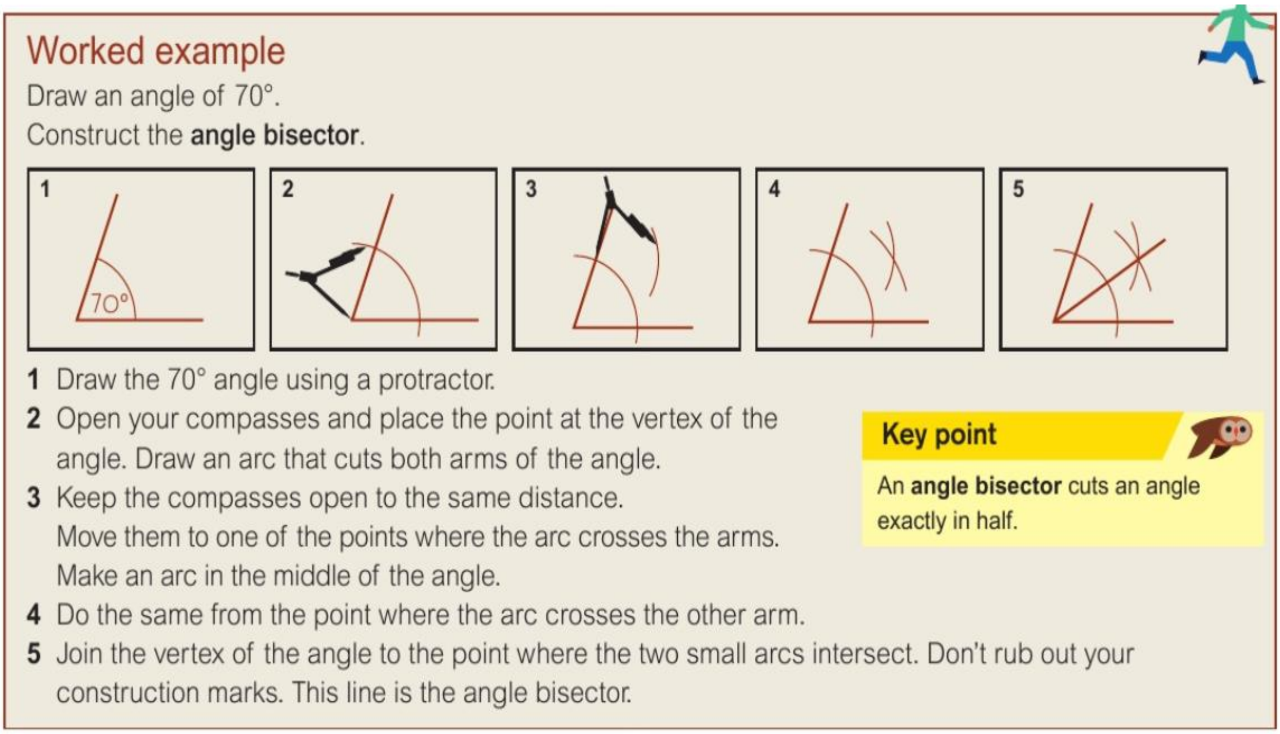
Construct triangle ABC where AB = 6 cm, AC = 5 cm, and angle BAC = 60°. Then bisect angle BAC.

Solution Outline:

- Draw a line AB = 6 cm.
- Use a protractor to measure and draw a 60° angle at point A.
- Mark point C 5 cm from A on the angle line.
- Join points B and C to complete the triangle.
- Follow the angle bisecting steps above to bisect angle BAC.

Top Tips for Success:

- Always check that your compass width stays the same when needed.
- Use faint construction lines, then darken final answers.
- Practice the steps multiple times to gain accuracy.



Worked example

Draw an angle of 70°.
Construct the **angle bisector**.

1 Draw the 70° angle using a protractor.

2 Open your compasses and place the point at the vertex of the angle. Draw an arc that cuts both arms of the angle.

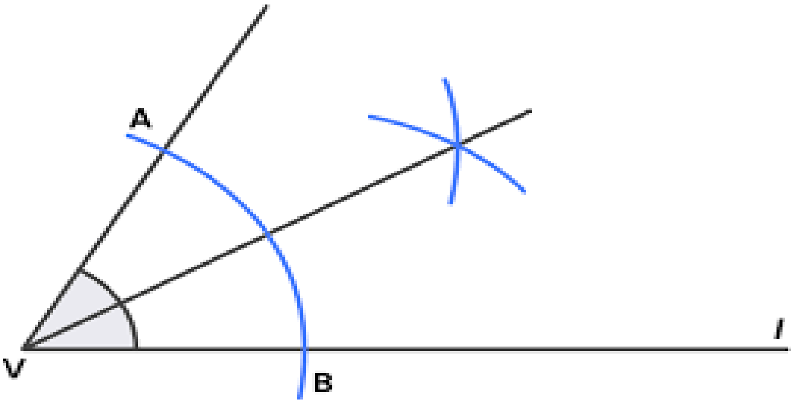
3 Keep the compasses open to the same distance. Move them to one of the points where the arc crosses the arms. Make an arc in the middle of the angle.

4 Do the same from the point where the arc crosses the other arm.

5 Join the vertex of the angle to the point where the two small arcs intersect. Don't rub out your construction marks. This line is the angle bisector.

Key point

An **angle bisector** cuts an angle exactly in half.



References

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