

# MPA International School



YEAR 7

MATHEMATICS

Reference Resources Booklet (1)

Name : .....

ID : .....

2026-2027 Academic Year

YEAR 7

MATHEMATICS

Reference Resources Booklet (1)

Unit – 1

Unit – 2

Unit – 3

Unit – 4

## Preface

At MPA International School, we are committed to nurturing learners who are not only knowledgeable but also capable of guiding their own learning journey. This booklet has been carefully prepared to support our students as a confident, curious, and independent learner by providing clear, structured notes that reinforce key concepts and offer guidance across all related topics.

Each section in this booklet connects directly to the topics of the textbook, offering:

- Clear explanations of key ideas
- Concept summaries for quick revision
- Supportive notes that encourage **self-study** and **personal reflection**

This resource is designed to help students’ **review at their own pace**, explore topics more deeply, and strengthen what they’ve learned in class. Whether they’re preparing for a quiz, completing homework, or simply curious to know more—this booklet is here to guide and support them. Most importantly, use it to grow as a **self-directed learner**—someone who learns with purpose, confidence, and curiosity.

This resource is not meant to replace active learning or classroom discussion but to empower students to revisit important content at their own pace—whether reviewing after a lesson, preparing for a quiz, or exploring further out of curiosity.

We hope this booklet empowers you to take ownership of your learning with purpose and pride.

**Academic Team**

**MPA International School**

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## Unit (1) Analysing and Displaying Data

### 1.1 Averages and ranges

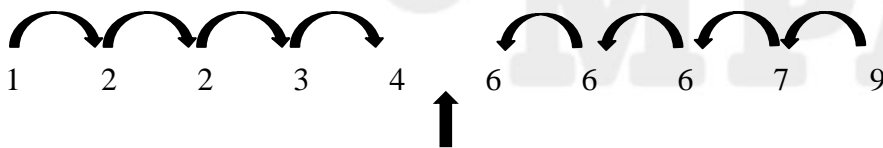
<b>Mode</b>	The mode is the most <b>frequent</b> value.	<ul style="list-style-type: none"> <li>- Count how many of each value appear.</li> <li>- The mode is the value that appears the most.</li> <li>- You can have more than one mode.</li> </ul>	<u>2,2</u> , 5, 6, 7, 8	The mode is <b>2</b>
<b>Mean</b>	The mean is the <b>average</b> or norm.	<ul style="list-style-type: none"> <li>- Add up all the values to find a total.</li> <li>- Divide the total by the number of values you added together.</li> </ul>	$2 + 2 + 5 + 6 + 7 + 8$ $= 30$ $30 \div 6 = 5$	The mean number is <b>5</b>
<b>Median</b>	The median is the <b>middle</b> number.	<ul style="list-style-type: none"> <li>- Put the numbers in order from the smallest to the largest.</li> <li>- Cover up one number on each end until you get to the middle.</li> </ul>	2, 2, 5, <u>6</u> , 7, 8, 9	The median is <b>6</b>
<b>Range</b>	The range is the <b>difference between</b> the highest value and the lowest value.	<ul style="list-style-type: none"> <li>- Find the highest and lowest values.</li> <li>- Subtract the lowest value from the highest.</li> </ul>	<u>2</u> , 2, 5, 6, 7, <u>8</u> $8 - 2 = 6$	The range is <b>6</b>



*Hey diddle diddle  
 The median's the middle  
 You add and divide for the mean.  
 The mode is the one you see the most  
 and the range is the difference  
 between.*

**Worked example**

Find the median of 4, 2, 6, 7, 2, 1, 3, 6, 6, 9



**median = 5**

- There are two middle values. The median is **halfway** between 4 and 6.

The “**modal**” value is another way of saying “**the mode.**”

## 1.2 More averages and ranges

### Discrete and Continuous Data

Key characteristics of **Discrete** Data:

- The data can be counted.
- It is impossible to divide the values.
- The data cannot be measured.
- There are only a few possible values.

#### Examples of Discrete data



Number of books



Number of people



Number of Billiard balls

Key characteristics of **Continuous** Data:

- Continuous variables are not counted.
- The data is measurable.
- An infinite number of possible values.
- Histograms are used to represent data graphically.

#### Examples of Continuous Data




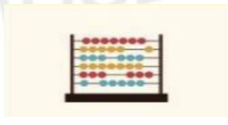


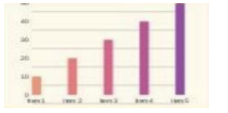


Weight of a new born baby



Body temperature



Speed of a horse

Points	Discrete Data	Continuous Data	
<i>Meaning</i>	Discrete data has clear spaces between values.	Continuous data falls on a continuous sequence.	
<i>Can you count the data?</i>	Yes, the data is usually units counted in whole numbers.	Generally, NO	
<i>Can you measure the data?</i>	NO	YES	
<i>Values</i>	It has a finite number of possible values. The values cannot be divided into smaller pieces and cannot have additional meaning.	It has an infinite number of possible values within an interval. The values can be subdivided into smaller and smaller pieces.	
<i>Graphical Representation</i>	Bar Chart	Histogram	
<i>Examples</i>	<ul style="list-style-type: none"> <li>• The number of students in a class.</li> <li>• The number of workers in a company.</li> <li>• The number of parts damaged during transportation.</li> <li>• Shoe sizes.</li> <li>• A number of languages an individual speaks.</li> <li>• The number of home runs in a baseball game.</li> <li>• The number of test questions you answered correctly.</li> </ul>	<ul style="list-style-type: none"> <li>• The amount of time required to complete a project.</li> <li>• The height of children.</li> <li>• The amount of time it takes to sell shoes.</li> <li>• The amount of rain, in inches, that falls in a storm.</li> <li>• The square footage of a two-bedroom house.</li> <li>• The weight of a truck.</li> <li>• The speed of cars.</li> <li>• Time to wake up.</li> </ul>	 

## Grouped frequency table

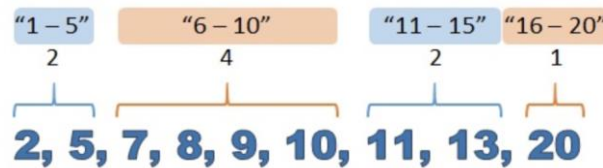
A grouped frequency table is a ***frequency table*** where the numbers are grouped.

It shows you how often numbers within each group appear in a list of numbers. Imagine you had a set of numbers:

**2, 10, 5, 13, 20, 11, 7, 8, 9**

A grouped frequency table is a way to present this data more simply:

Group	Frequency
1 - 5	2
6 - 10	4
11 - 15	2
16 - 20	1



We see that 2 numbers appear in the **1 - 5** group (2 and 5). We write **1 - 5** in the **Group** column.

We write 2 in the **Frequency** column because numbers in the **1 - 5** range appear 2 times in the list of numbers.

Group	Frequency
1 - 5	2

We see that there are 4 numbers in the **6 - 10** group (7, 8, 9, and 10). We write **6 - 10** in the **Group** column.

We write 4 in the **Frequency** column because numbers in the **6 - 10** range appear 4 times in the list of numbers.

Number	Frequency
1 - 5	2
6 - 10	4

The cumulative frequency table is complete when all numbers and their frequencies have been entered:

Group	Frequency
1 - 5	2
6 - 10	4
11 - 15	2
16 - 20	1

The groups of numbers in the lists are in the **Group** column.

How often each number appears in each group is in the **Frequency** column.

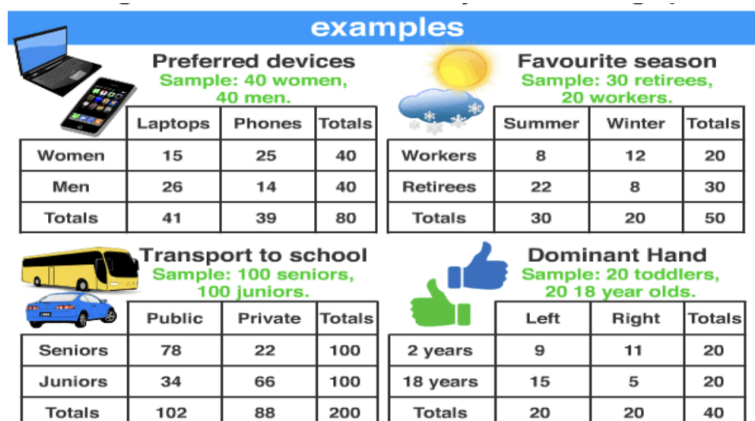
## Modal Class

- The modal class is the one with the ***highest frequency***.

### 1.3 Two-way tables and bar charts

#### Two-way table

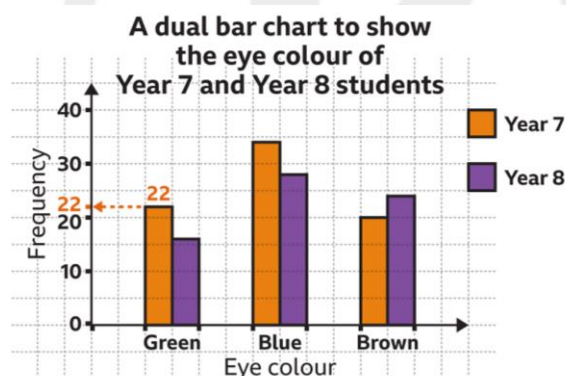
Two-way tables are a way of sorting data so that the frequency of each category can be seen quickly and easily.



#### Dual bar chart

A double bar graph is a **graphical representation of two sets of data on the same graph.**

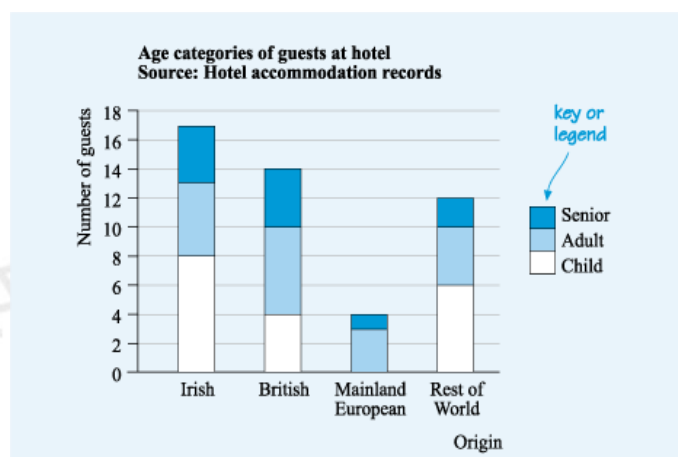
A double bar graph is a **graphical representation of information showing two bars beside each other of various heights.**



#### Compound bar charts

A compound bar chart is a graph that **combines two or more types of information in one chart.**

It can also compare different quantities. A compound bar chart is a type of bar chart where columns can be split into sections to show a breakdown of data.

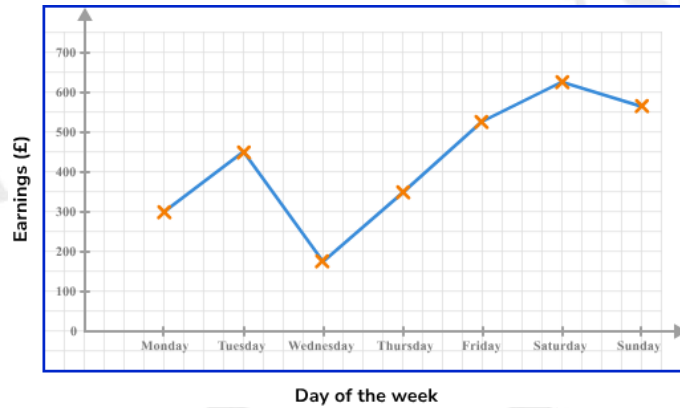


## 1.4 More Graphs and tables (1)

### Line graph

A line graph - also known as a line plot or a line chart - is **a graph that uses lines to connect individual data points**. A line graph displays quantitative values over a specified time interval.

In finance, line graphs are commonly used to depict the historical price action of an asset or security.



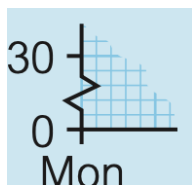
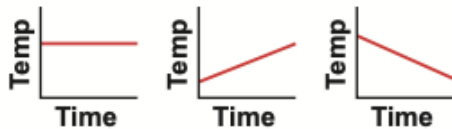
### How to construct a line graph?

- Step 1: Identify the variables. ...
- Step 2: Determine the variable range. ...
- Step 3: Determine the scale of the graph. ...
- Step 4: Number and label each axis and title the graph.
- Step 5: Determine the data points and plot on the graph. ...
- Step 6: Draw the graph.

### Types of line graphs

Which graph shows the temperatures

- increasing
- decreasing
- staying the same?



The zig-zag shows there's a break in the axis scale.

## Unit (2) Number

### 2.1 Rules of divisibility

Ways to determine if one number can be evenly divided by another, without actually dividing them.

<b>Number</b>	<b>Rule</b>	<b>Example</b>	<b>Divisible?</b>
<b>2</b>	The last digit is even: (0,2,4,6,8).	5 7 <b>6</b> is even	<b>YES</b>
		8 3 <b>1</b> is not even	<b>NO</b>
<b>3</b>	The sum of the digits is divisible by 3.	<b>5 3 7</b> --- $5+3+7 = 15$ , $15 \div 3 = 5$	<b>YES</b>
		<b>8 3 3</b> --- $8+3+3 = 14$ , $14 \div 3$ has a remainder	<b>NO</b>
<b>4</b>	Double the tens digit and add to the ones digit. This answer is divisible by 4.	2 5 <b>7 6</b> --- $2 \times 7 + 6 = 20$ , $20 \div 4 = 5$	<b>YES</b>
		2 5 <b>2 5</b> --- $2 \times 2 + 5 = 9$ , $9 \div 4$ has a remainder	<b>NO</b>
<b>5</b>	The last digit is 0 or 5.	3 8 <b>5</b> last digit is 5	<b>YES</b>
		9 6 <b>4</b> last digit is 4	<b>NO</b>
<b>6</b>	The number is divisible by both 2 and 3.	<b>4 3 2</b> --- even; $4+3+2 = 9$ and $9 \div 3 = 3$	<b>YES</b>
		<b>5 2 4</b> --- even; $5+2+4 = 11$ and $11 \div 3$ has a remainder	<b>NO</b>
<b>7</b>	Double the last digit and subtract it from the rest of the number. This answer is 0 or divisible by 7.  * You can apply this rule to that answer again.	<b>5 8 1</b> --- $2 \times 1 = 2$ , $58 - 2 = 56$ and $56 \div 7 = 8$	<b>YES</b>
		<b>3 2 1</b> --- $2 \times 1 = 2$ $32 - 2 = 30$ and $30 \div 7$ has a remainder	<b>NO</b>

<b>8</b>	The last three digits are divisible by 8.	<b>4 5 1 4 4</b> $144 \div 8 = 18$	<b>YES</b>
		<b>3 2 2 4 5</b> $245 \div 8$ has a remainder	<b>NO</b>
<b>9</b>	The sum of the digits is divisible by 9.	<b>4 8 5 1</b> --- $4+8+5+1 = 18$ $18 \div 9 = 2$	<b>YES</b>
		<b>2 6 1 3</b> --- $2+6+1+3 = 12$ $12 \div 9$ has a remainder	<b>NO</b>
<b>10</b>	The last digit is 0.	<b>3 4 7 0</b> --- last digit is 0	<b>YES</b>
		<b>8 3 2 5</b> --- last digit is 5	<b>NO</b>
<b>11</b>	Alternate subtracting and adding the digits from left to right. This answer is 0 or divisible by 11.	<b>9 5 1 1 7</b> $9-5+1-1+7 = 11$ $11 \div 11 = 1$	<b>YES</b>
		<b>9 3 8 2</b> $9-3+8-2 = 12$ $12 \div 11$ has a remainder	<b>NO</b>
<b>12</b>	The number is divisible by both 3 and 4.	<b>8 3 5 2</b> --- $8+3+5+2 = 18$ , $18 \div 3 = 6$ and $2 \times 5 + 2 = 12$ , $12 \div 4 = 3$	<b>YES</b>
		<b>7 8 1 6</b> --- $7+8+1+6 = 22$ $22 \div 3$ has a remainder and $2 \times 1 + 6 = 8$ , $8 \div 4 = 2$	<b>NO</b>

## 2.2 Factors, multiples, and primes

### What you need to know:

#### Multiples and factors

**Multiples:** The result of multiplying a number by an integer. It is the times table of a number.

Multiples of 4: 4, 8, 12, 16, 20 ...  
 Multiples of 5: 5, 10, 15, 20, 25, ...

Multiples are the list of times tables.

**Factors:** A number that divides exactly into another number without a remainder. It is often helpful to write them in pairs.

Write them in pairs first so you don't miss any!



Factors of 20 = 1, 2, 4, 5, 10, 20

#### Prime numbers

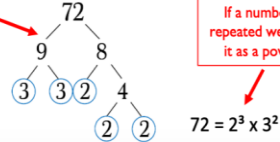
**Prime:** This is a number that has exactly 2 factors; 1 and itself.

2 is the only even prime. The first 10 prime numbers are:  
 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

These are not the only prime numbers.

**Prime factorisation:** This is when we split a number into its prime factors using a factor tree. We circle the prime factors.

We need to find pairs of numbers that multiply to give the number above.



If a number is repeated we write it as a power.

$$72 = 2^3 \times 3^2$$

### Key Terms:

**Prime number:** A prime is a number that has only two factors which are 1 and itself.

**Multiple:** A number in the given numbers times table.

**Factor:** A number that fits into another number exactly.

**LCM:** The smallest number that is in the times tables of the given numbers.

**HCF:** The biggest number that divides exactly into two or more numbers.

### You need to be able to:

- Identify factors and multiples.
- Identify a prime number.
- Complete a prime factor tree and write the number in index notation.
- Calculate HCF and LCM of two values using an appropriate method.

### Hegarty maths clip numbers

Factors and Multiples: 27 and 33

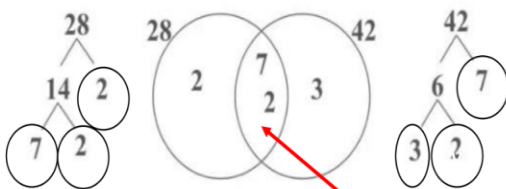
Prime Numbers and HCF and LCM: 28 – 32, 34 – 36



### What you need to know:

#### HCF

This is where we find the biggest number that divides exactly into two or more numbers.



The prime numbers (the circled numbers) go in the venn diagram.

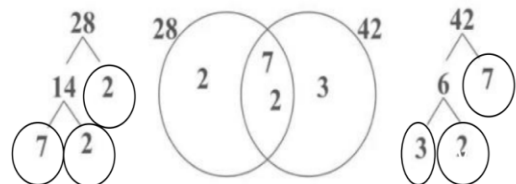
To calculate the HCF we multiply all of the numbers in the intersection.

The HCF of 28 and 42 =  $7 \times 2 = 14$

Remember numbers that appear in both prime factor tree's go in the middle of the venn diagram.

#### LCM

This is where we find the smallest number that appears in the given numbers times tables.



There are 2 different ways of calculating the LCM:

Method 1:

Multiply all of the numbers in the venn diagram =  $2 \times 7 \times 2 \times 3 = 84$

Method 2:

Multiply the large number outside the venn diagram by the small numbers in the opposite circle:

$28 \times 3 = 84$  or  $42 \times 2 = 84$

All 3 calculations give you the same answer so you could do all 3 to check your answer is correct.

The LCM of 28 and 42 = 84

## 2.3 Positive and Negative Numbers

### What you need to know:

#### Ordering positive and negative numbers

The further left you go the smaller the number is.  
Example - Put the following numbers in ascending order

2 -4 5 -2 3 -1

Ascending - smallest to biggest = -4, -2, -1, 2, 3, 5

#### Calculating with positive numbers

When you are unable to complete a calculation mentally use a written method. The most common method for addition, subtraction and multiplication is column method; for division use Bus Stop method.

Examples

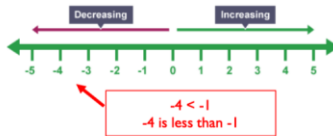
$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 8121 \\ - 457 \\ \hline 475 \end{array}$$

$$\begin{array}{r} 24 \\ \times 16 \\ \hline 240 \\ 144 \\ \hline 384 \end{array}$$

$$5 \overline{) 15.8} \\ \underline{7^2} 9.4 \\ \underline{9.0} \\ 40$$

When adding or subtraction start with the ones column.



#### Powers of 10

Use a place value grid to help if needed. If there are any gaps before the decimal point, you need to fill them in with zeros.

Remember:

To multiply by 10, move the digits on a place value grid left by one column. To multiply by 100, move two columns, multiply by 1000 3 columns etc. To divide by 10, move the digits on a place value grid right by one column. To divide by 100, move two columns, divide by 1000 3 columns etc.

Examples:

$$53 \times 10 = 530$$

$$18 \times 100 = 1800$$

$$60 \times 10^3 = 60000$$

$$53 \div 10 = 5.3$$

$$18 \div 100 = 0.18$$

$$60 \div 10^3 = 0.060$$

$$10^3 = 1000$$

### Key Terms:

**Ascending** - smallest to biggest

**Descending** - biggest to smallest

**Approximate** - to estimate a number, amount or total often using rounding to make them easier to deal with

**Integer** - a WHOLE number that is positive or negative

**Negative** - a number less than 0 with a minus sign

**Place Holder** - we use 0 as a place holder

**Place Value** - The value of a digit depending on its place in the number

### You need to be able to:

- Use and order positive and negative numbers
- Use the symbols  $<$ ,  $>$  and understand the  $\neq$  symbol
- Add, subtract, multiply and divide positive and negative numbers
- Multiply and divide by powers of 10
- Round numbers to a given power of 10
- Understand the order of operations

### Hegarty maths clip numbers

Positive Integers: 1-12, 18-24

Place value and powers of ten: 13-17

Negative Numbers: 37-44



### Reminder:

#### Inequality signs

$<$  less than  
 $>$  Greater than  
 $=$  equal to  
 $\neq$  not equal to

### What you need to know:

#### Directed Numbers - positive and negative numbers

#### Adding and Subtracting

Remember:

**Subtract** when two different signs appear next to each other

**Add** when two of the same signs appear next to each other

Examples:

$$3 - 7 = -4$$

$$-2 - 9 = -11$$

$$-5 + 2 = -3$$

$$5 - 2 = 7$$

$$-4 + 5 = -9$$

$$-8 - 2 = -6$$

$$5 + 2 = 7$$

$$-4 - 5 = -9$$

$$-8 + 2 = -6$$

You can draw and use a number line to help you with adding and subtracting



In the last three examples the two signs appear next to each other

#### Multiplying and Dividing

Remember:

When the signs are **different** the answer is **negative**

When the signs are the **same** the answer is **positive**

Examples:

$$5 \times -4 = -20$$

$$-3 \times -8 = 24$$

$$-6 \times 2 = -12$$

$$45 \div -5 = -9$$

$$-100 \div -10 = 10$$

$$-18 \div 9 = -2$$

#### Tip

Always check your answers by using rounding and inverse operations

Example:  $9 \times 20 = 180$   
Check  $180 \div 20 = 9$

#### Rounding to the nearest power of ten

5495 to the nearest 1000

5475 to the nearest 100

5475 to the nearest 10

5000

5400

5470

6000

5500

5480

To round to the nearest 10, 100 or 1000 look at the digit in the corresponding column. Look at the next digit. 5 or more 'round up' (increase by 1) 4 or less 'round down' (keep the same) Fill any spaces with zeros.

Round 4,853 to the nearest 10, 100 and 1,000.

Round 76,982 to the nearest 10, 100 and 1,000.

4853 to the nearest 10 is 4,850

4853 to the nearest 100 is 4,900

4853 to the nearest 1,000 is 5,000

76982 to the nearest 10 is 76,980

76982 to the nearest 100 is 77,000

76982 to the nearest 1,000 is 77,000

#### Order of Operations - BIDMAS

1<sup>st</sup> Brackets

2<sup>nd</sup> Indices

3<sup>rd</sup> Division

4<sup>th</sup> Multiplication

5<sup>th</sup> Addition

6<sup>th</sup> Subtraction

This is the order in which you should work out your calculations. Any brackets work them out first. Then any indices. Then division and multiplication (in the order they appear). Finally any addition and subtraction (in the order they appear)

Examples:

$$4 + 2 \times 3$$

$$4 + 6 = 10$$

$$3 + (7 \times 2 - 5)$$

$$3 + 9 = 12$$

$$(8 - 2) \times 3^2$$

$$6 \times 3^2$$

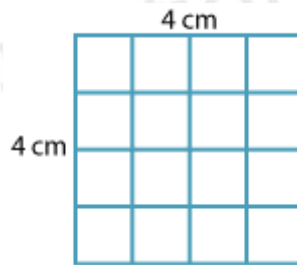
$$6 \times 9 = 54$$

## 2.4 Squares and square roots

### Squares

The **square** of a number is the number multiplied by itself. For example, four squared,  $4^2$ , is  $4 \times 4 = 16$ . This can be shown as a diagram.

A square of side length 4 cm has an area of  $4 \text{ cm} \times 4 \text{ cm} = 16 \text{ cm}^2$ .



The **square root** of a number is the number that when multiplied by itself gives the original number.

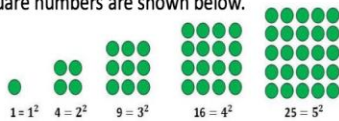
To illustrate:

$6 \times 6 = 36$ , so six is the square root of 36. We write  $\sqrt{36} = 6$ .

#### What you need to know:

##### Squares, cubes and roots

**Square numbers:** This is when we multiply a number by itself, the first 5 square numbers are shown below.



**Square roots:** This is the number that we started with to get the square numbers.

$\sqrt{49} = 7$  because  $7 \times 7$  is 49

Remember the answer is 7 not  $7 \times 7$ .

$\sqrt{100} = 10$  because  $10 \times 10$  is 100

**Cube numbers:** This is when we multiply a number by itself and then by itself again, the first 5 cube numbers are shown below.



##### Index form

**Index number:** An index number is a number which is raised to a power. The power, also known as the index, tells you how many times you have to multiply the number by itself.

$2^5$  is the index notation.

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

#### Key Terms:

**Square:** A square number is the result of multiplying a number by itself.

**Cube:** A cube number is the result of multiplying a number by itself twice.

**Root:** A root is the reverse of a power.

**Indices:** These are the squares, cubes and powers.

**Operation:** In maths these are the functions  $\times \div + -$ .

#### You need to be able to:

- Recognise and calculate square numbers and roots.
- Recognise and calculate cube numbers and roots.
- Complete calculations with a mixture of powers and roots.
- Use BIDMAS to complete calculations.
- Use the laws of indices to simplify expressions.

#### Hegarty maths clip numbers

Powers and Roots: 99 – 102

Laws of Indices: 105, 106 and 110



## 2.5 More powers and roots

You can use *index notation* to write a number to a *power* or *index*. The power tells you how many times the number is *multiplied by itself*.

$$2^3 = 2 \times 2 \times 2$$

$2^3$  is "2 to the power 3." 3 is the power.

To find the **cube** of a number, multiply it by itself and then multiply by the number again.

$$2 \text{ cubed} = 2 \times 2 \times 2$$

Cube Numbers from 1 to 50

$1^3 = 1$	$11^3 = 1331$	$21^3 = 9261$	$31^3 = 29791$	$41^3 = 68921$
$2^3 = 8$	$12^3 = 1728$	$22^3 = 10648$	$32^3 = 32768$	$42^3 = 74088$
$3^3 = 27$	$13^3 = 2197$	$23^3 = 12167$	$33^3 = 35937$	$43^3 = 79507$
$4^3 = 64$	$14^3 = 2744$	$24^3 = 13824$	$34^3 = 39304$	$44^3 = 85184$
$5^3 = 125$	$15^3 = 3375$	$25^3 = 15625$	$35^3 = 42875$	$45^3 = 91125$
$6^3 = 216$	$16^3 = 4096$	$26^3 = 17576$	$36^3 = 46656$	$46^3 = 97336$
$7^3 = 343$	$17^3 = 4913$	$27^3 = 19683$	$37^3 = 50653$	$47^3 = 103823$
$8^3 = 512$	$18^3 = 5832$	$28^3 = 21952$	$38^3 = 54872$	$48^3 = 110592$
$9^3 = 729$	$19^3 = 6859$	$29^3 = 24389$	$39^3 = 59319$	$49^3 = 117649$
$10^3 = 1000$	$20^3 = 8000$	$30^3 = 27000$	$40^3 = 64000$	$50^3 = 125000$

### Perfect Cube

A perfect cube is a number that is obtained by multiplying the same integer three times. For example, multiplying the number 4 three times results in 64. Therefore, 64 is a perfect cube. Therefore, the perfect cube = number  $\times$  number  $\times$  number. The cube root of 64 is 4. A number is said to be a perfect cube if it can be decomposed into a product of the same three numbers.

### How to Find the Perfect Cube?

The perfect cube of a number can be checked by following the steps given below:

- **Step 1:** Prime factorize the given number starting from the smallest prime number (2).
- **Step 2:** Once the **prime factorization** is done, club every three same factors together.
- **Step 3:** Repeat the step for all the sets of the group of the same three factors. If any factors are left behind and do not fit into a group of three same factors, then the given number is not a perfect cube. Otherwise, the given number is a perfect cube.

## Perfect Cube example

2	216
2	108
2	54
3	27
3	9
3	3
	1

Prime Factorization of 216 is

$$\underbrace{2 \times 3 \times 2 \times 3 \times 2 \times 3}$$

$$\therefore 216 = \underbrace{6 \times 6 \times 6}$$

Same number is multiplied thrice.

So, 216 is a perfect cube.

2	200
2	100
2	50
5	25
5	5
	1

Prime Factorization of 200 is

$$\underbrace{2 \times 5 \times 2 \times 5 \times 2}$$

$$\therefore 200 = 10 \times 10 \times 2$$

Since all the three factors are not the same, 200 is not a perfect cube.

## Cube Root

The **cube root** of a number is that number that is multiplied 3 times to get the original number.

The inverse of a cube is the **cube root**.  $2^3 = 8$ , so the cube root of 8 is  $\sqrt[3]{8} = 2$ .



Cubes and Cube Roots



## Cube Root 1 to 20



$\sqrt[3]{1} = 1$	$\sqrt[3]{11} = 2.2239$
$\sqrt[3]{2} = 1.2599$	$\sqrt[3]{12} = 2.2894$
$\sqrt[3]{3} = 1.4422$	$\sqrt[3]{13} = 2.3513$
$\sqrt[3]{4} = 1.5874$	$\sqrt[3]{14} = 2.4101$
$\sqrt[3]{5} = 1.7099$	$\sqrt[3]{15} = 2.4662$
$\sqrt[3]{6} = 1.8171$	$\sqrt[3]{16} = 2.5198$
$\sqrt[3]{7} = 1.9129$	$\sqrt[3]{17} = 2.5712$
$\sqrt[3]{8} = 2$	$\sqrt[3]{18} = 2.6207$
$\sqrt[3]{9} = 2.08$	$\sqrt[3]{19} = 2.6684$
$\sqrt[3]{10} = 2.1544$	$\sqrt[3]{20} = 2.7144$

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## 2.6 Calculations

### Order of Operations

You may have learnt a few different acronyms to remember the order of operations, including **BODMAS** and **PEDMAS**. **BIDMAS** stands for **B**rackets, **I**ndices, **D**ivision and **M**ultiplication, **A**ddition, and **S**ubtraction.

Example:

$$\begin{aligned} & 35 \div 7 + 3^2 - (4 + 6) \\ & = 35 \div 7 + 3^2 - 10 \text{ Brackets } (4 + 6) = 10 \\ & = 35 \div 7 + 9 - 10 \text{ Indices } 3^2 = 9 \\ & = 5 + 9 - 10 \text{ Division } 35 \div 7 = 5 \\ & = 14 - 10 \text{ Addition and subtraction from left to right} \\ & = 4 \end{aligned}$$

One common mistake is often made when re-entering a number. The answer button, or Ans can be used to recall the last answer, and this can then be used for the final calculation.

#### **More Involved Calculations**

Use BIDMAS to help you.

**Example:**  $\frac{23.5 \times 7.73}{0.497}$  or  $(23.5 \times 7.75) \div 0.497$

$$\begin{aligned} \text{estimate} & \approx \frac{20 \times 8}{0.5} \\ & = \frac{160}{0.5} \\ & = 320 \end{aligned}$$

$$320 \times 1.08 = \text{£}345.60$$

$$100\% - 17\% = 83\% = 0.83$$

$$850 \times 0.83 = \text{£}705.50$$

$$\text{Calculation: } 23.5 \times 7.73 = 181.655$$

$$181.655 \div 0.497 = 365.50$$

Use the **Ans** key to recall a previous answer.

**Example:**  $\frac{23.5 - 7.73}{4 \times 0.6}$

$$\begin{aligned} \text{estimate} & \approx \frac{20 - 8}{2} \\ & = \frac{12}{2} = 6 \end{aligned}$$

$$\text{Calculation } (23.5 - 7.73) = 15.77$$

$$(4 \times 0.6) = 2.4$$

$$= \frac{15.77}{2.4}$$

$$= 15.77 \div 2.4 = 6.57$$

**Make sure you are familiar with BIDMAS with a basic calculator. A scientific calculator will calculate this for you.**

## Unit (3) Equations, functions, and formulae

### 3.1 Simplifying algebraic expressions

- What is a **variable**?

In Maths, a variable is an **alphabet** that represents an unknown number or unknown value or unknown quantity. The variables are specially used in the case of algebraic expressions or algebra.

For example,  $x + 9 = 4$  is a linear equation where  $x$  is a variable, where 9 and 4 are constants.

**Variable**

**Variable** : A letter that represents an unknown number.      eg.  $x, a, n$

$$1 + x = 3$$

↑  
**variable**

- What is a **term**?

In Algebra a term is either a single number or variable, or numbers and variables multiplied together.

Terms are separated by + or – signs, or sometimes by divide.

*Expression*

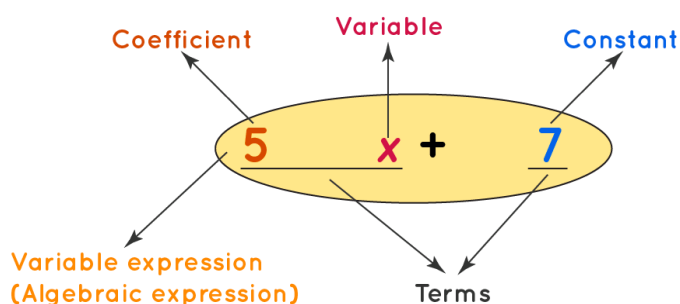
$$\frac{4x - 7}{\text{Terms}} = \frac{5}{\text{Terms}}$$

*Terms*

- What is an **expression**?

An expression in maths is a sentence with a minimum of two numbers or variables and at least one maths operation. This maths operation can be addition, subtraction, multiplication, or division.

### What is an Algebraic Expression



<b>expression</b>	<b>equation</b>
does not have an equal sign (only + or -)	has an equal sign separating two expressions

Expression vs Equation

Expressions	Equations
$5y + 3$	$5y + 3 = 8$
$2xy - 5y + 3$	$2xy - 5y = 3xy$
$5 - 7 + 4 \times 2$	$5 - 7 + 4 \times 2 = 6$


- What are Like Terms?

In Algebra, the like terms are defined as the terms that contain the same variable which is raised to the same power. In algebraic terms, only the numerical coefficients can vary.

Like And Unlike Algebraic Terms	
Like Terms	Unlike Terms
$2x + 19x$	$2x + 19a$
$4w - 10w$	$4w - 10w^2$
$14.2r - 12r$	$12r - 12s$
$32a^2 + 9a^2$	$32a^2 + 9a^3$
$8y + 5y$	$8y + 5$

### Collecting Like Terms


**Collecting like terms** is a way of simplifying algebraic expressions. To do this we identify the like terms in an algebraic expression and combine them by adding or subtracting.

 **Example** Collect the like terms  $3a + 4b + 2a - 2b$


$3a$  and  $+2a$  are like terms  
 $+4b$  and  $-2b$  are also like terms, but they are different to the terms with the letter  $a$ . The plus or minus sign in front of a term belongs to that term.

$$3a + 4b + 2a - 2b = 3a + 2a + 4b - 2b$$


$$= 5a + 2b$$

**Key point** 


**Like terms** contain the same letter (or do not contain a letter). You simplify an expression by collecting like terms.

**Key point** 

$2 \times 2 \times 2 = 2^3$   
 In the same way, you can write  $b \times b \times b = b^3$

**Key point** 

Write letters in alphabetical order.  
 Write numbers before letters.  
 $a \times 2 = 2 \times a = 2a$

**Key point** 

The identity symbol ( $\equiv$ ) shows that two expressions are *always* equivalent.  
 For example,  $a + 2b \equiv 2b + a$ .



## Worked example

Simplify

a  $3b \times 2b$

$$3b \times 2b = 3 \times b \times 2 \times b$$

$$= 3 \times 2 \times b \times b$$

$$= 6b^2$$

The order of multiplication does not matter.

b  $\frac{8b}{4}$

$$\frac{8b}{4} = 2b$$

$\frac{8b}{4}$  means  $8b \div 4$ . Work out  $8 \div 4$ .

Term	Definition	Picture/Example
<b>Terms</b>	Quantities that you <b>ADD</b> to form an algebraic expression are called terms.	There are 3 terms in  $4n + 6b - 8$  The terms are:
<b>Like Terms</b>  You can <b>COMBINE</b> Like Terms <b>**COMBINE</b> means add, so use the addition rules (SSS, DSD)	terms with the <b>same variable</b> raised to the <b>same power</b>  You <b>CAN</b> add/subtract like terms.	
<b>Unlike Terms</b>	terms whose variables are not the same, or who have the same variable, but it's raised to a different power  You <b>CANNOT</b> add/subtract unlike terms.	

### 3.2 Writing algebraic expressions

An *expression* is a set of algebraic terms combined using +, −, ×, or ÷, such as  $4y + 8z^2$ .

#### **Example 1**

Anya is  $x$  years old. Jade is 2 years older than Anya. Bethan is three times as old as Jade.

Write a simplified expression for the sum of their ages.

Anya's age is already given as  $x$  years old.

Jade is Anya's age plus 2:  $(x + 2)$  years old.

Bethan is three times Jade's age:  $3(x + 2) = (3x + 6)$  years old.

The sum of their ages is  $x + x + 2 + 3x + 6 = (5x + 8)$  years old.

#### **Example 2**

A rectangle has a width of  $x$  cm. Its height is 2cm longer than its width. Write an expression for the perimeter of the rectangle.

The perimeter is found by adding the lengths of the sides together.

The width is already given as  $x$  cm. The height is 2cm longer, so the height is  $(x + 2)$  cm.



### 3.3 STEM: Using formulae

- A **formula** is a general rule for a relationship between quantities. You use a formula to work out an **unknown** quantity by substituting.

#### **Worked example**

The **formula** used to calculate speed is: speed =  $\frac{\text{distance}}{\text{time}}$

Work out the speed of a cyclist who travels 1000 metres in 20 seconds.

$$\begin{aligned} \text{Speed} &= \frac{1000}{20} \\ &= 50 \text{ m/s} \end{aligned}$$

Substitute the values into the formula.  
Write the units.

m/s means metres per second.

### 3.4 Writing Formulae

**Expression:** A mathematical statement written using **symbols, numbers, or letters**, e.g.  $3x + 2$  or  $5y^2$ .

**Equation:** A statement showing that **two expressions are equal**. Example -  $2y - 17 = 15$


**Identity:** An equation that is **true for all values** of the variables.

An identity uses the symbol:  $\equiv$ . Example;  $2x \equiv x + x$

**Formula:** Shows the relationship between two or more variables.

Example: Area of a rectangle = length  $\times$  width or  $A = L \times W$

**Simplifying Expressions:** Collect 'like terms'. Be careful with negatives.  $x^2$  and  $x$  are not like terms.

**Worked example** 

Storing furniture in a warehouse costs \$12 per week.  
Write a formula for the cost,  $C$ , of storing furniture for  $y$  weeks.

$12y$  ————— Write down the cost each week.  
Multiply the cost by the number of weeks.

$C = 12y$  ————— Write  $C =$  your expression.

---

### 3.5 Brackets and formulae

**Expand:** To expand a bracket, multiply each term in the bracket by the expression outside the bracket.

Example:  $3(m + 7) = 3x + 21$

- **Expand** a bracket means multiply every number inside the bracket by the number or letter outside the bracket.

## Unit (4) Fractions

### 4.1 Working with fractions

#### What is a fraction?

Fractions represent the parts of a whole or collection of objects. A fraction has two parts. The number on the top of the line is called the numerator. It tells how many equal parts of the whole or collection are taken. The number below the line is called the denominator. It shows the total number of equal parts the whole is divided into or the total number of the same objects in a collection.

- The larger denominator does not mean a larger fraction.

#### Equivalent fractions

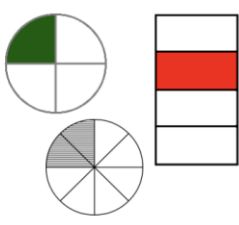
Two fractions are equivalent if one is a multiple of the other.

- To find the equivalent multiply / divide the numerator and denominator by the same number.

**Key Concept**

$\frac{1}{4}$

$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16}$



= 0.25

**Examples**

Simplify  $\frac{3}{24}$  a fraction

$\frac{3}{24} \xrightarrow{+3} \frac{1}{8}$

---

Write 0.34 as

1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	•	3	4	

$= \frac{34}{100} = \frac{17}{50}$

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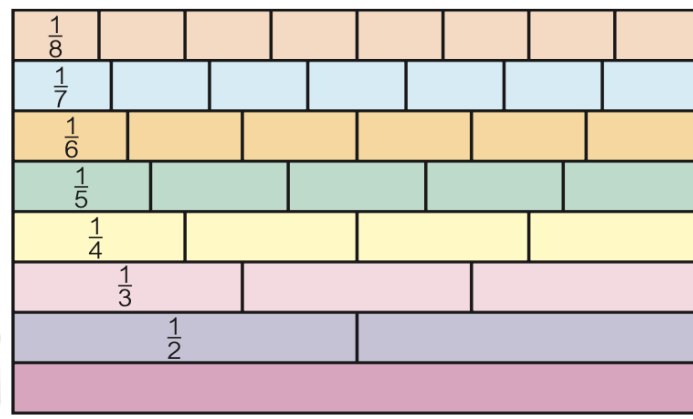
What is  $\frac{3}{5}$  of 45

$45 \div 5 \times 3 = 27$

Write  $\frac{6}{7}$  as a decimal

$\frac{6}{7} \rightarrow 7 \overline{) 6.00000}$

#### The fraction wall



- Integer** : Whole number.
- Ascending Order** : Place in order, smallest to largest.
- Descending Order** : Place in order, largest to smallest.

**Inequality** : Statement that contains  $<$  or  $>$  to show which quantity is greatest/smallest.

**Examples**

To order these fractions, make the denominators the same.	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{2}$
	$\frac{6}{8}$ (4)	$\frac{3}{8}$ (2)	$\frac{4}{8}$ (5)
To order these, convert them all to decimals.	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$
	56%	0.871	23%
	0.56 2	0.75 3	0.23 1
	23%	56%	$\frac{6}{7}$
		$\frac{3}{4}$	0.857... 4
		$\frac{6}{7}$	0.871

Comparing fractions

$\frac{3}{8}$   $\times$   $\frac{4}{5}$   
 $3 \times 5 < 4 \times 8$   
 $15 < 32$   
 $\frac{3}{8} < \frac{4}{5}$

Comparing Fractions Using Cross Multiplication

Improper Fractions

**improper fractions**

The numerator is the same or bigger than the denominator.

numerator  $\frac{5}{2}$  denominator

$\frac{2}{2}$

$\frac{4}{4}$

$\frac{7}{4}$

$\frac{5}{3}$

$\frac{8}{5}$

Mixed numbers

**Key Concept**

Mixed numbers  
These are made up of a whole number and a fraction.

$4\frac{3}{5}$   
 $= \frac{4 \times 5 + 3}{5}$   
 $= \frac{23}{5}$

Converting improper fractions to mixed numbers

Example: Convert  $\frac{10}{3}$  to a mixed fraction.

$\frac{10}{3} = 10 \div 3 = 3 \text{ R } 1$   
 $= 3\frac{1}{3}$

Answer:  $3\frac{1}{3}$

## Converting mixed numbers to improper fraction

Example: Convert  $2\frac{1}{9}$  to an improper fraction.

$$2\frac{1}{9} + \quad \xrightarrow{2 \times 9 + 1} \quad \frac{19}{9}$$

## Fraction of amounts:

Divide the bottom (denominator), and multiply by the top (numerator)

$$\frac{3}{4} \text{ of } 36$$

Divide by the denominator then multiply by the numerator

$$36 \div 4 = 9 \times 3 = 27$$

$$\left( \frac{3}{4} \text{ of } 36 = 27 \right)$$

## 4.2 Adding and Subtracting Fractions

### Adding or subtracting fractions with the same denominators

$$\frac{3}{5} + \frac{1}{5} = \boxed{?}$$

$$\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$$

$$\frac{7}{12} - \frac{2}{12} = \boxed{?}$$

$$\frac{7}{12} - \frac{2}{12} = \frac{7-2}{12} = \frac{5}{12}$$

### Adding or subtracting fractions with different denominators

$$\frac{1}{2} + \frac{3}{10} = \boxed{?}$$

- ① Find LCD and create equivalent fractions

$$LCD = 10$$

$$\frac{1}{2} = \frac{5}{10} \quad \text{and} \quad \frac{3}{10} = \frac{3}{10}$$

- ② Add the new fractions

$$\frac{5}{10} + \frac{3}{10} = \frac{5+3}{10} = \frac{8}{10}$$

- ③ Reduce fraction to simplest terms

$$GCF = 2 \quad \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$$

$$\frac{10}{3} - \frac{9}{5} = \boxed{?}$$

① Find LCD and create equivalent fractions

$$LCD = 15 \quad \frac{10}{3} \xrightarrow{\times 5} \frac{50}{15} \quad \text{and} \quad \frac{9}{5} \xrightarrow{\times 3} \frac{27}{15}$$

② Subtract new fractions

$$\frac{50}{15} - \frac{27}{15} = \frac{50 - 27}{15} = \frac{23}{15}$$

③ Change improper fraction to a mixed number

$$\frac{23}{15} = 1 \frac{8}{15}$$

### 4.3 Fractions, Decimals and Percentages

#### Key Vocabulary

- Fraction : Represents part of a whole number
- Decimal : A number with digits after the decimal point
- Percentage : Means “out of 100”

#### Conversion of FDP

- (1) Fraction to Decimal : **Divide** the numerator by the denominator  
(Numerator ÷ Denominator)
- (2) Decimal to Fraction :
  - (i) Explore the **hidden denominator**
  - (ii) **Multiply** the numerator and denominator with **10, 100, 1000 ...** (it depends on the **decimal place ...**)
  - (iii) **Simplify** the calculation
- (3) Fraction to Percentage : **Multiply** the fraction with **100%**
- (4) Decimal to Percentage : **Multiply** by **100%**
- (5) Percentage to Fraction : Write as a fraction **over 100** and **simplify**
- (6) Percentage to Decimal : **Divide** by 100

<b>Fraction</b>	$\frac{1}{100}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
<b>Decimal</b>	0.01	0.1	0.125	0.2	0.25	0.3	0.5	0.6	0.75
<b>Percentage</b>	1%	10%	12.5%	20%	25%	33.3%	50%	66.6%	75%

#### Converting Decimals to Fractions

To convert a decimal to a fraction, use **place value**. The first number after the decimal place is worth tenths, the next is worth hundredths, the next thousandths and so on.  
e.g. What is 0.7 as a fraction?

Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandths
		0	.	7		

The 7 represents tenths, so this can be written as 7/10. This fraction cannot be simplified or **cancelled down**, so it is the final answer.

#### Converting Decimals to Percentages

Once a number is written as a decimal, it is easy to convert it to a percentage. Remember that 'per cent' means 'per hundred', so converting from a decimal to a percentage can be done by **multiplying by 100** (move the decimal place two places to the right).

e.g. 0.375 as a percentage  
=  $0.375 \times 100 = 37.5\%$

#### Converting Fractions to Decimals

To convert a fraction to a decimal, you need to use **written division methods** to divide the numerator by the denominator.

e.g. Convert 3/8 into a decimal.  
Divide 3 by 8.  
 $\frac{3}{8} = 3 \div 8 = 0.375$

#### Converting Percentages to Decimals and Fractions

To convert from a percentage, first convert it to a decimal and then convert the decimal to a fraction if required.

e.g. Convert 24% to a decimal and a fraction.  
24% = 0.24 (2 tenths and 4 hundredths)

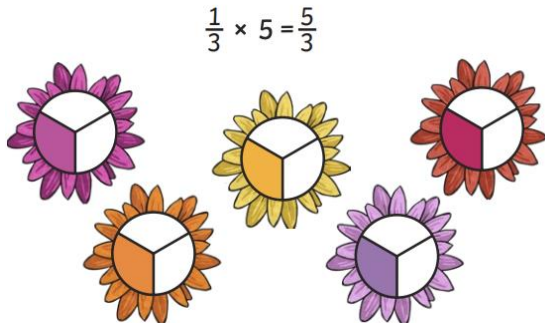
Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandths
		0	.	2	4	

0.24 as a fraction  
is 24/100 which simplifies to 6/25 (simplified by removing a common factor of 4).

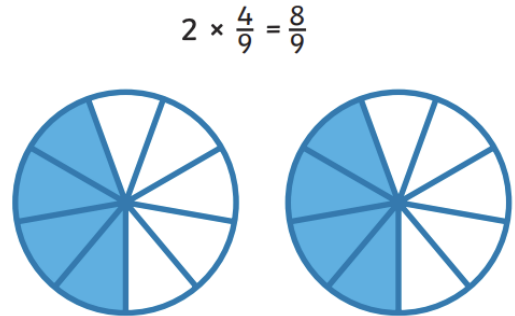
#### 4.4 Multiplying by a fraction

#### 4.5 Working with mixed numbers

Multiply Unit Fractions by an Integer



Multiply Non-Unit Fractions by an Integer



Multiply Mixed Numbers by Integers

Convert to an improper fraction and multiply the numerator by the integer.

$$2\frac{1}{4} \times 2 = \frac{9}{4} \times 2 = \frac{18}{4} = 4\frac{2}{4} = 4\frac{1}{2}$$

Use repeated addition.

$$2\frac{1}{4} \times 2 = 2\frac{1}{4} + 2\frac{1}{4} = 4\frac{2}{4} = 4\frac{1}{2}$$

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#### References

<https://www.twinkl.com/mm/>

<https://qualifications.pearson.com/>